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Margin Requirements with Intraday Dynamics

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Margin requirements with intraday dynamics[1]

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Abstract

Both in practice and in the academic literature, models for setting margin requirements in futures markets use daily closing price changes. However, financial markets have recently shown high intraday volatility, which could bring more risk than expected. Such a phenomenon is well documented in the literature on high-frequency data and has prompted some exchanges to set intraday margin requirements and ask intraday margin calls. This article proposes to set margin requirements by taking into account the intraday dynamics of market prices. Daily margin levels are obtained in two ways: first, by using daily price changes defined with different time-intervals (say from 3 pm to 3 pm on the following trading day instead of traditional closing times); second, by using 5-minute and 1-hour price changes and scaling the results to one day. An application to the FTSE 100 futures contract traded on LIFFE demonstrates the usefulness of this new approach.

Keywords: ARCH process, clearinghouse, exchange, extreme value theory, futures markets, high-frequency data, intraday dynamics, margin requirements, model risk, risk management, stress testing, value at risk.

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1. Introduction

The existence of margin requirements decreases the likelihood of customers' default, brokers' bankruptcy and systemic instability of futures markets. Margin requirements act as collateral that investors are required to pay to reduce default risk. [4] Margin committees face a dilemma however in determining the magnitude of the margin requirement imposed on futures traders. On the one hand, setting a high margin level reduces default risk. On the other hand, if the margin level is set too high, then the futures contracts will be less attractive for investors due to higher costs and decreased liquidity, and finally less profitable for the exchange itself. This quandary has forced margin committees to impose investor deposits that represent a practical compromise between meeting the objectives of adequate prudence and liquidity of the futures contracts.

For products traded on the London International Financial Futures and Options Exchange (LIFFE), margin requirements are set by the London Clearing House (LCH)[5] using the London Systematic Portfolio Analysis of Risk (SPAN) system, a specifically developed variation of the SPAN system originally introduced by the Chicago Mercantile Exchange (CME). The London SPAN system is a non-parametric risk-based model that provides output of margin requirements that are sufficient to cover potential default losses in all but the most extreme circumstances. The inputs to the system are estimated margin requirements relying on price movements that are not expected to be exceeded over a day or couple of days. These estimated values are based on diverse criteria incorporating a focus on a contract's price history, its close-to-close and intraday price movements, its liquidity, its seasonality and forthcoming price sensitive events. Market volatility is specially a key factor to set margin levels. Most important however is the extent of the contract's price movements with a policy for a minimum margin requirement that covers three standard deviations of historic price volatility based on the higher of one-day or two-day price movements over the previous 60-day trading period. This is

[4] Futures exchanges also use capital requirements and price limits to protect against investor default.

[5] The LCH risk committee made up of qualified risk management members is responsible for all decisions relating to margin requirements for LIFFE contracts. Margin committees generally involve experienced market participants who have widespread knowledge in dealing with margin setting and implementation, through their exposure to various market conditions and their ability to respond to changing environments (Brenner (1981)). The LCH risk committee is independent from the commercial function of the Clearinghouse.

akin to using the Gaussian distribution where multiples of standard deviation cover certain price movements at various probability levels.[6]

The academic literature has applied a number of alternative statistical approaches in order to compute the margin requirement that adequately protects against default at various probability levels and/or determine the probabilities associated with different margin requirements. Figlewski (1984) and Gay *et al* (1986) classically assume that futures price movements follow a Gaussian distribution. One well-documented problem with using a particular distribution such as the Gaussian distribution is model risk. In particular, it is well known that the Gaussian distribution underestimates in most cases the weight of the tails of the distribution. Longin (1996) uses extreme value theory to quantify this statement and shows that the empirical distribution of financial asset price changes is fat-tailed while the Gaussian distribution is thin-tailed. Edwards and Neftci (1988) and Warshawsky (1989) use the historical distribution of past price changes which overcomes the underestimation issue of assuming normality. However, the historical distribution is unable to deal with very low probability levels due to the lack of sufficient price changes available for analysis.

A distinct approach focuses on an economic model for broker cost minimization in which the margin is endogenously determined (Brennan (1986)). Another approach developed by Craine (1992) and Day and Lewis (1999) is based on the fact that the distributions of the payoffs to futures traders and the potential losses to the futures clearinghouse can be described in terms of the payoffs to barrier options. Initial margins requirements can then be related to the present value of such options.

Kofman (1993), Longin (1995 and 1999), Booth *et al* (1997) and Cotter (2001) apply extreme value theory, a statistical theory that specifically models the tails of the distribution of futures price changes. This latter framework specifically focuses on the main measurement issue relating to margin setting, namely, trying to adequately model quantiles and probabilities of the distribution tails for future price changes. As the problem of setting margin requirements is related to the tails of the distribution of futures price changes (the left tail for a long position and the right tail for a short position) it is beneficial to examine specifically lower and upper tail percentiles. Extreme value theory does exactly this by focusing only on tail values thereby minimising model risk that is associated with procedures that model the full distribution of

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[6] For instance, under the hypothesis of normality for price movements, two standard deviations would cover 97.72% of price movements, and three standard deviations 99.87%.

futures price changes. Extreme value theory removes the need for making assumptions of the exact distributional form of the random process under analysis as the limiting distribution of extreme price changes is the same for many classes of distributions and processes used to describe futures price changes (see Longin and Solnik (2001)). Another advantage of the extreme value approach is the parametric form that allows one to extrapolate to out-of-sample time frames unlike the use of the historical distribution of price changes that is constrained to in-sample predictions. By having an objective likelihood function we avoid the problem of subjectively defined stress tests that try to examine the impact of financial crises. Furthermore, extreme value theory requires tail estimates that are time invariant due to their fractal nature. This allows for precise tail measurement incorporating a simple and efficient scaling law for different frequency intervals, for example from intraday to daily estimates.

One question that we may ask about the nature of risk management is whether the clearinghouse should care more about ordinary market conditions or more about extraordinary market conditions. In other financial institutions such as banks two distinct approaches are used: value at risk models for ordinary market conditions and stress testing for extraordinary market conditions (see Longin (2000)). The clearinghouse must also address both sets of market conditions in margin setting so as to minimize the likelihood of investor default by examining a range of probabilities of price movements associated with common and uncommon events. The first approach is conditional reflecting the changing of market conditions over time while the second approach is unconditional trying to incorporate extreme events that occurred over a long period of time. All studies above are based on unconditional distributions and cannot reflect current market conditions. Cotter (2001) considers a conditional process by applying a GARCH specification to address issues relating to the dynamic features of futures contracts volatility.

Previous studies based on statistical models used closing prices to estimate daily margin requirements mainly due to data unavailability. However, trading on futures markets takes place on an intraday level and a complete understanding of their operations requires analysis of high-frequency intraday features (see Cotter (2004)). Margin setting using intraday dynamics incorporates the full information set regarding price movements over the trading day. In contrast, margin setting using closing prices only uses trading information around close of day. Intraday dynamics are important. For instance, it is well documented that daily volatility varies over time with particular characteristics (Bollerslev *et al* (1992)). However, more recently, intraday volatility has also been examined and distinct patterns are also documented. For example, macroeconomic announcements impact volatility sharply but their impacts have a life span of less than two hours, and thereafter have a negligible influence on price movements (Bollerslev *et al* (2000)). Thus an analysis of daily prices alone would not take account of these intraday activities.

Intraday price movements supply the margin setter with a mechanism to adequately describe and predict the impact of futures price volatility within the appropriate timeframe. In terms of statistical modelling the impact of futures volatility on margin requirement setting require a certain minimum number of observations for first accurately identifying the empirical feature, next developing a model that adequately describes the feature and finally testing the model to predict market occurrence. Notwithstanding this, the clearinghouse must ensure that they are modelling the same economic event in their analysis of financial data. For instance, futures price changes may exhibit a structural change over time from say the 1980s to the 1990s. Thus given the average lifespan of many futures contracts is one year margin setting is based on analysis of price movements for this interval size in this paper. However, in model development, this interval size may sometimes provide insufficient observations at daily frequency using various statistical techniques. Using higher frequency intraday price changes and scaling to relatively low frequency daily estimates overcomes this modelling difficulty.

In practice clearinghouses are beginning to recognize the importance of intraday dynamics. For example, in 2002, the LCH has introduced an additional intraday margin requirement that is initiated if price movements on a contract challenge the prevailing margin requirement. Specifically, an intraday margin requirement is initiated if a contract price changes by 65% of the margin requirement originally set for that contract. In this case, the Clearinghouse requires an additional margin payment for falling prices on a long position or for rising prices on a short position. The possible impact of intraday price movements is now clearly, and rightly so, of concern to risk management overseers for LIFFE contracts.

The main contribution of this paper is to take into account the intraday dynamics of futures market prices by computing margin requirements. All previous academic studies considered daily closing prices only, thus missing important information. Closing prices alone lose information regarding price movements and their associated transaction activity within the trading day. The clearinghouses modelling margin requirements should incorporate the intraday price movements in margin setting. Daily margin levels are obtained in two ways: first, by using daily price changes defined with different time-intervals (say from 3 pm to 3 pm on the following trading instead of traditional closing times); second, by using 5-minute and 1-hour price changes and scaling the results to one day following Dacarogna *et al* (1995). As shown by

Merton (1980) for risk measures (as opposed to performance measures), it is beneficial to use data with the highest frequency in order to get more precise estimates of the tail parameter. In our paper, different statistical distributions are also used to model futures price changes: the Gaussian distribution, the extreme value distribution and the historical distribution. An ARCH-type process is also used to take into account the time-varying property of financial data. An application is given for the FTSE 100 futures contract traded on LIFFE.

The remainder of the paper is organized as follows. The statistical models used for the distribution of futures contract price changes and the scaling method are presented in the next section. Section 3 provides a description of the FTSE 100 futures contract data used in the application and a detailed statistical analysis of the intraday dynamics of the market prices. Section 4 presents empirical results for margins by taking into account the intraday dynamics. Finally, a summary of the paper and some conclusions are given in Section 5.

2. Statistical models and scaling method

This section presents the different statistical models used to compute the margin level for a given probability. It also presents the scaling method to obtain daily margin levels from intraday price changes.

2.1 The extreme value distribution

The theoretical framework applied in this study relies on the findings of extreme value theory. According to this statistical theory three types of asymptotic distribution can be obtained: Gumbel, Weibul and the one of concern to this study, the Fréchet distribution, which is obtained for fat-tailed distributions (see Gnedenko (1943)). Weak convergence is assumed to occur for the Fréchet distribution underpinned by the maximum domain of attraction (MDA). This allows for approximation to the characteristics of the Fréchet distribution giving rise to a semi-parametric estimation procedure. This theoretical framework offers a number of advantages to margin setting. First, the main prudence issue in determining margin requirements is to protect against default that results from extreme price movements. These price changes are extreme values and as such should be modeled with procedures specifically focused on capturing these quantile and probability estimates, and this is exactly what extreme value does. Second, modeling only the tail of the distribution as opposed to the center of the distribution, which is irrelevant for margin setting, minimizes bias in the estimation procedure. Third, tail behavior of the fat-tailed Fréchet distribution exhibits a self-similarity property that allows for an easy extension for multi-period margin estimation using a simple scaling rule.

Examining the framework and begin by assuming that a margin requirement can be measured as futures price change, represented by a random variable, *R*, and that exceeding this level is estimated at various probabilities. Furthermore, assume that the random variable is independent and identically distributed (iid) and belonging to the true unknown cumulative probability density function F_R.[7] We are interested in the probability that the maximum of the first n random variables exceeds a certain price change, r, [8]

(1) $$P\{M_n > r\} = 1 - F^n(r)$$

for n random variables, $M_n = \max\{R_1, R_2, ..., R_n\}$.

The probability estimator could also be expressed as a quantile where one is examining what margin requirement is sufficient to exceed futures price changes at various probability levels.[9]

Whilst the exact distribution is unknown, assuming the distribution exhibits the regular variation at infinity property, then asymptotically it behaves like a fat-tailed distribution.

(2) $$1 - F^n(r) \approx a r^{-\alpha}$$

where a represents the scaling parameter and α the shape parameter.[10]

This expression is for any given frequency and it is easy to extend the framework to lower frequencies as these extremes have an identical tail shape. For instance, taking the single period price changes, R, and extending these to a multi-period setting, kR, using the additive property of a fat-tailed distribution from Feller's theorem (Feller (1971)):

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[7] The successful modelling of financial returns using GARCH specifications clearly invalidates the iid assumption. De Haan *et al* (1989) examine less restrictive processes more akin with futures price changes only requiring the assumption of stationarity and this is followed in this paper.

[8] Extreme value theory is usually detailed for upper order statistics focusing on upper tail values and the remainder of the paper will follow this convention. This study also examines empirically the lower order statistics focused on lower tail values.

[9] For the issue at hand the probability of exceeding a predetermined margin level on a short position for n price changes is: $P_{short} = P\{M_n > r_{short}\} = \delta$, where r_{short} represents the margin level on a short position and δ is the unknown exceedance probability given by $1 - F^n(r)$.

[10] The shape parameter α is related to the tail index τ often used in the EVT literature by the relation: $\alpha = 1/\tau$.

(3) $$1-F^{n}(kr)\approx kar^{-\alpha}$$

Importantly the shape parameter, α, remains invariant to the aggregation process and also has implications for empirical benefits in its actual estimation.[11] Dacarogna *et al* (1995) have shown that high-frequency tail estimation has efficiency benefits due to their fractal behavior. In contrast, low frequency estimation suffers from negative sample size effects. Furthermore for ease of computation, the scaling procedure does not require further estimation, but only involves parameters from the high-frequency analysis, shown to provide the most detailed information on futures price movements.

The regular variation at infinity property represents the necessary and sufficient condition for convergence to the fat-tailed extreme value distribution. Thus it unifies fat-tailed distributions and allows for unbounded moments:

(4) $$\lim_{t\to+\infty}\frac{1-F_R(t\cdot r)}{1-F_R(t)}=r^{-\alpha}$$

By l'Hopital's rule it can be shown that the student-t, and symmetric non-normal sum-stable distributions, and certain ARCH processes with an unconditional stationary distribution and even assuming conditionally normal innovations all exhibit this condition as their tails decline by a power function. Subsequently all these distributions exhibit identical behavior far out in the tails. In contrast, other distributions such as the normal distribution, and the finite mixtures of Gaussian distributions have a tail that declines exponentially which declines faster than a power decline and thus are relatively thin-tailed. The shape parameter, α, measures the degree of tail thickness and the number of bounded moments (see appendix for details of the semi-parametric estimation procedure). A shape parameter greater than 2 implies that the first two moments, the mean and variance, exist whereas financial studies have cited value between 2 and 4 suggesting that not all moments of the price changes are finite (see Longin (1996)). In contrast, support for the Gaussian distribution would require a shape parameter equal to infinity, as all moments exist. Thus the estimate of the shape parameter distinguishes between different distributions and for instance, α represents the degrees of freedom of the Student-*t* distribution and equals the characteristic exponent of the sum-stable distribution for $\alpha < 2$.

[11] The α-root scaling law for the extreme value estimates is similar in application to the √ scaling procedure of a normal distribution.

Given the asymptotic relationship of the random variable to the fat-tailed distribution, non-parametric tail estimation takes place giving two related mechanisms for describing the margin estimates. The first focuses on the margin requirement and determines the probability of various price movements, r_p:

(5) $$r_p = r_t (m / np)^{1/\alpha}$$

By using this estimate we can examine different margin requirements that would not be violated at various probability levels and implicitly determine if the trade-off between optimizing liquidity and prudence is being met. Rearranging gives the probability, p, of exceeding any preset margin requirement:

(6) $$p = (r_t / r_p)^{\alpha} m / n$$

Again these probabilities are used to determine if the prudence and commercial concerns of the futures exchange is reached.

2.2 The APARCH process

To model the time-varying behavior of price changes suggested by the previous analysis, we use the Asymmetric Power ARCH (APARCH) developed by Ding *et al* (1993). This model nests many extensions of the GARCH process. As well as encompassing three ARCH specifications (ARCH, Non-linear ARCH and Log-ARCH), two specifications of the GARCH model (using standard deviation and variance of returns), it also details two asymmetric models (both ARCH and GARCH versions). It is given by:

(7) $$\sigma_t^d = \alpha_o + \sum_{i=1}^{p} \alpha_i \left(\left| \varepsilon_{t-i} \right| + \gamma_i \varepsilon_{t-i} \right)^d + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^d$$

for $\alpha_0, \alpha_i, \beta_j \geq 0, \quad \alpha_i + \beta_j \leq 1, \quad -1 \leq \gamma_i \leq 1.$

The APARCH incorporates volatility persistence, β, asymmetries, γ, and flexibility of power transformations, d, in the estimation of volatility. Detailing the model, the process presents the volatility measure in the form of a Box-Cox transformation whose flexibility allows for different specifications of the residuals process. This transformation provides a linear representation of non-linear processes. As well as describing the traditional time dependent volatility feature, the model specifically incorporates the leverage effects, γ, by letting the autoregressive term of the conditional volatility process be represented as asymmetric absolute residuals. A general class of volatility models incorporating the non-linear versions are defined by the power coefficient, d.

The APARCH(1, 1) is applied to the price series at the end of the sample during December 2000. A number of variations of the model are applied and Akaike's (AIC) and Schwarz's (BIC) selection criteria are used to determine the best fitted process. Fat-tails are accounted for by assuming that the conditional distribution is a Student-*t* distribution.

3. Data analysis

3.1 Data

The empirical analysis is based on transaction prices for the FTSE 100 futures contract trading on the LIFFE exchange (data are obtained from *Liffedata*). This exchange has made a clear distinction, between contracts that are either linked to an underlying asset or developed formally on the basis of links to the recently developed European currency, the euro, and those that remain linked to factors outside the currency area. The FTSE 100 represents the most actively traded example of the latter asset type.

Data are available on the stock index contract for four specific delivery months per year, March, June, September and December. Prices are chosen from those contracts with delivery months on the basis of being the most actively traded using a volume crossover procedure. The empirical analysis is completed for sampling frequencies of 5 minutes, 1 hour and 1 day. The first interval is chosen so as to meet the objective of analyzing the highest frequency possible and capturing the most accurate risk estimates but also avoids microstructure effects such as bid ask effects. For the daily frequency, the price changes are computed by taking different starting (and ending) times to define the day: the beginning of the "day" can start from 9 am (the opening of the trading day) to 5 pm (the closing of the trading day). Nine different time-series of daily price changes are then obtained. Log prices (or log prices to the nearest trade available) for each interval are first differenced to obtain each period's price change. The period of analysis is for the year 2000 involving 247 full trading days corresponding to an average life span of an exchange traded futures contract. The FTSE 100 futures daily interval encompasses 113 5-minute intervals and nine hourly intervals. A number of issues arise in the data capture process. First, all holidays are removed. This entails New Year's (2 days), Easter (2 days), May Day (1 day), spring holiday (1 day), summer holiday (1 day), and Christmas (2 days). In addition, trading took place over a half day during the days prior to the New Year and Christmas holidays and these full day periods are removed from the analysis.

3.2 Basic statistics

Basic statistics are reported in Table 1 for price changes (Panel A) and for squared price changes (Panel B). Concentrating on the first four moments of the distribution we study their behavior according to frequency of measurement. Most predominately the kurtosis increases as the frequency increases. For price changes, the (excess) kurtosis is equal to 0.26 for a 1-day frequency, 1.54 for a 1-hour frequency and 254.50 for a 5-minute frequency. The high kurtosis (higher than the value equal to 0 implied by normality) gives rise to the fat-tailed property of futures price changes. It is also illustrated by the probability density function and QQ plots of the shapes of price changes for different frequencies given in Figure 1. The extent of fat-tails is strongest for 5-minute realizations supporting the summary statistics. Also, the magnitude of values for these realizations can be very large as indicated by the scale of the density plots. These features generally result in the formal rejection of a Gaussian distribution using the Kolmogorov-Smirnov test.[12] Deviations from normality are strongest at the highest frequency. The other moments emphasize the magnitude and scale of the realizations sampled at different frequencies. On average, price changes were negative during the year 2000 and unconditional volatility increases for interval size. Selected quantiles reinforce divergences in magnitude at different frequencies. Similar conclusions can be made for the proxy of volatility, the squared price changes, although the skewness and kurtosis are more pronounced.

Notwithstanding the divergence in moments for different frequencies, it is interesting to examine daily price changes and volatility as it is these estimates that are used in the statistical analysis resulting in daily margin requirements. In addition to examining daily price changes using closing prices that are the norm in margin setting through the marking to market system, daily price changes are also defined with different time-intervals. Basic statistics are reported in Table 2 and a time-series plot for two of these time-intervals, using opening prices and closing prices are presented in Figure 2. Whilst the mean price changes remain reasonably constant, other moments are more diverging. For instance, skewness goes from -0.09 to -0.47 and the kurtosis statistic goes from being platykurtic (-0.32) to leptokurtic (1.52). Also the dispersion of various quantiles is considerable. Again inferences for the squared price changes are similar

[12] Whilst a formal rejection of normality for the full distribution of daily price is not recorded at common significance levels the tail behaviour in Figure 1 clearly indicates a fat-tailed property.

although greater in magnitude. However it can be observed that both time-series have similar time-varying features evidencing volatility clustering with periods of high and low volatility but the diverging features are clearly demonstrated as suggested by the magnitude of realizations.

Given the divergence indicated by the intraday analysis, it is interesting to incorporate these features in the margin setting process.

3.3 Extreme value analysis

Shape parameter estimates using different time-intervals to compute daily price changes are presented in Table 3 for the left tail (Panel A) and the right tail (Panel B). The point estimates are calculated using the weighted least squares technique that minimizes the small sample bias following Huisman *et al* (2001). The point estimates range from 2.57 to 6.34 and the values are generally in line with previous findings (see Cotter (2001)). As the shape parameter is positive, the extreme value distribution is a Fréchet distribution that is obtained for a fat-tailed distribution of price changes.

We also use the shape parameter estimates to test if the second and the fourth moment of the distribution are well defined. For classical confidence level (say 5%), we are unable to reject the hypothesis that the variance is infinite in any scenario, whereas we are able to reject the hypothesis that the kurtosis is infinite in many scenarios. Advantageously the extreme value scaling law is applicable as it only requires the existence of a finite variance.

3.4 Conditional estimation

Time-varying behavior is described from fitting the APARCH model to daily price changes from different time-intervals at the end December 2000. The fat-tailed property is accounted for by assuming the error innovations belong to a Student-*t* distribution. The APARCH estimates consistently indicate that the conditional distributions exhibit persistence, with for example, past volatility impacting on current volatility as is typical of GARCH modeling at daily intervals.[13] Furthermore the conditional distributions vary according to the time intervals analyzed that will give rise to different margin requirements.

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[13] For instance the parameter estimates based on closing prices are: $\alpha_0 = 0.014$, $\alpha_1 = 0.011$, $\beta_1 = 0.962$, $\gamma_1 = -0.999$ and $d = 1.855$. Further details and coefficient estimates are available on request.

4. Model-based margin requirements

This section presents empirical results for margin requirements obtained with daily price changes (4.1) and 5-minute and 1-hour price changes scaled to one day (4.2).

4.1 Margin requirement based on daily price changes

Table 4 presents margin requirements obtained with daily price changes for a long position (Panel A) and for a short position (Panel B). Margin requirements are computed for a given probability. Four different values are considered: 95%, 99%, 99.6% and 99.8% corresponding to average waiting periods of 20, 100, 250 and 500 trading days. Thinking of risk management for financial institutions, probabilities of 95% and 99% would be associated with ordinary adverse market events modeled by value at risk models, and probabilities of 99.6% and 99.8% with extraordinary adverse market events considered in stress testing programs. In the margin setting context, the probability reflects the degree of prudence of the exchange: the higher the probability, the higher the margin level, the less risky the futures contract for market participants, but the less attractive the contract for investors. Margin requirements are also computed with various statistical models: three unconditional distributions (Gaussian, extreme value and historical) and a conditional process (the Asymmetric Power ARCH process).

For the presentation of the results, the extreme value distribution will be the reference model as it presents many advantages (parametric distribution, limited model risk, limited event risk) and as the problem of margin setting is mainly concerned with extreme price changes. Beginning with the analysis of extreme value estimates, we first note that variation occurs in the estimates based on the different time-intervals to define daily price changes. For example, for a long position and a probability level of 95%, the estimated margin level ranges from 1.83% to 2.05% of the nominal position. For the most conservative level of 99.8%, it ranges from 2.77% to 5.32%, almost double. Also there does not seem to be a systematic pattern to these deviations. For instance, for a probability of 95%, the minimum is obtained with 2 pm prices and the maximum for closing prices, and for a probability of 99.8%, the minimum is obtained with 3 pm prices and the maximum for 10 am prices. The same remarks apply to a short position. These findings suggest that the daily price change distributions vary to some extent based on different time-intervals sampled suggesting separate tail behavior for each price series.

Turning to the estimates obtained under normality, some key insights are obtained. First, the measures are almost identical for long and short positions due to the assumption of a symmetric distribution of futures price changes and an average price change close to zero over the period considered. In contrast, the extreme value distribution and the historical distribution take account of the possibility of non-symmetric features in line with the oft cited stylized facts of financial time series, and verified for the FTSE 100 futures contract of diverging upper and lower distribution shapes. However, in line with all the estimates, diverging margin estimates occur according to the time-intervals used to define price changes. For example, for a long position and a probability of 95%, the estimated margin varies from 1.83% using 3 pm prices to 2.05% using closing prices. Traditional comparisons of extreme value and normal risk estimates suggest the latter underestimates tail behavior due to its exponential tail decline that results in relatively thin-tailed features. These findings hold for the FTSE 100 contract for high probability levels of 99.6% and 99.8%. In contrast, for the relatively low probability level of 95%, this conclusion cannot be sustained and this is due to this confidence level representing a common rather than extreme threshold. For instance, the probability of this event occurring using daily data is once every 20 trading days representing a typical event rather than an extreme one, although it is the latter events that need to be guarded against to avoid investor default.

Then turning to the historical estimates, diverging margin requirements again occur according to the time-interval chosen with the largest (smallest) estimate on a long position at the 95% level happening at 1 pm (10 am). These estimates are based on using the historical price series gathered for the year 2000. The historical estimates are confined to in-sample inferences due to the limited number of price observations. This implies that margin setting using the historical distribution that tries to avoid investor default may not be able to model the events that actually cause the default, whereas in contrast, extreme value theory specifically models these tail values.

The margin requirements based on the unconditional distributions may be compared to the other estimates such as the conditional estimates using the APARCH process. Again it is clear that estimation at different time-intervals necessitates diverging margins. For instance, the out-of-sample estimates measured at 11 am and 1 pm (3 pm) represent the largest (smallest) possible margin requirements for a long position. Comparing the extreme value and APARCH estimates provides information on the distinction between unconditional and conditional environments facing margin setters. Distinct patterns occur based on the volatility estimation for the last trading day of the sample (December 29, 2000).

An alternative way to present the results is to compute the probability for a given margin level. Results for a large and a very large futures price change, ±5% and ±10%, are given in

Table 5. These results can be thought of as margin requirements that would be violated at certain probabilities. The results indicate a number of characteristics about the inherent risk in futures contracts. For instance, if a very large margin level of 10% is imposed, the probability of it being violated on any individual day is very low. For example, the probability of exceeding a price change of 10% for a long position using 10 am prices is 0.06 in contrast to 0.01 using closing prices. In terms of average waiting time-period these extreme price movements based on 10 am prices would occur approximately once every 15 years whereas in contrast, the occurrence for close of day prices is much less likely estimated at about every 103 years. Obviously the probability of exceeding a price movement increases as the price changes decrease so the likelihood of occurrence increases for 5% price moves. These results again imply that the starting point for the time interval used is an important factor in the setting of sufficient margin requirements as regardless of trading position there is a general finding that estimates taken using close of day prices are dominated by greater price movements at other intervals. In fact there is substantial variation in the excess probability estimates for different daily intervals.

4.2 Daily margin requirement based on high-frequency price changes

Table 6 presents daily margin requirements obtained with 5-minute and 1-hour price changes for a long position (Panel A) and for a short position (Panel B). Margin levels are scaled to one day (see Section 2 for the presentation of the scaling method) and compared to the ones obtained directly from daily price changes. The general lack of divergence of tail estimates for different frequencies supports the invariant with respect to aggregation property. Margin estimates are presented using the extreme value scaling procedure coupled with the average estimates based on daily estimates measured at different hourly intervals. Concentrating on the more extreme 99% level, the events that occur once every 100 trading days, the scaling procedure provides robust estimates in line with the average daily values.

5. Summary and conclusions

This paper proposes a method to incorporate the intraday dynamics of futures prices changes in daily margin setting thereby including lost information that is unavailable with the traditional approach of using closing prices in a marking to market system. The intraday futures price movements are relied on in two ways. First, daily prices movements and associated margins are measured using different time-intervals to define price changes, and second high-frequency 5-minute and 1-hour price changes are used to compute margins that are then scaled to give daily estimates.

Margin requirements by definition are collateral to avoid investor default, but must also be set by the Clearinghouse at a level that ensures the competitiveness of an exchange. This paper examines margin setting in the context of investor default through statistical analysis of extreme price movements. In practice margin setting for the FTSE 100 contract uses a customized version of the SPAN system developed by the CME. In particular, the minimum margin requirement incorporates implicitly the assumption of a Gaussian distribution for a contract's price movements as they must be able to match three standard deviations of price changes over the previous 60-day trading period.

Alternative statistical approaches are available for margin setting with varying degrees of attractiveness including assuming a Gaussian distribution, estimation based on the historical distribution of past price changes, conditional modeling with a GARCH process and unconditional estimation with extreme value theory. The key feature in separating out the approaches is to examine their ability in dealing with the fat-tailed characteristic of futures price movements. Model risk arises with any approach that assumes a particular distribution for price changes. For instance conditional estimation that incorporates the time-varying properties characteristic of financial price changes still requires assumptions for the conditional price generating process. Furthermore the supposition of normality incorporates a relatively thin-tailed distribution and leads to an underestimation of margin levels. The historical distribution of past price changes is incapable in dealing with the extreme price movements that result in investor default focusing only on in-sample probability levels. Finally, the approach advocated here using extreme value theory minimizes these problems by focusing exclusively on tail price movements thereby avoiding making inappropriate assumptions on a futures contract's price generating process, and also allowing for out-of-sample extrapolation. Advantageously this paper merges the theoretical benefits of extreme value theory to the empirical benefits of analyzing intraday dynamics that include scaling from high to low frequency margin levels.

After identifying the fat-tailed property of the futures price changes that becomes more pronounced for relatively high-frequency realizations, the paper identifies a number of key factors in margin setting. First and most important is the finding that intraday dynamics should be a key component in margin estimation. Daily price movements measured at different intervals can have a very tenuous relationship suggesting that the common procedure of using only close of day prices neglects the dynamics that investors actually face in trading futures. In addition using high-frequency intraday realizations negates this problem even if estimating at a daily frequency through a simple scaling law of extreme value theory. Second the paper illustrates the relative dominance of extreme value theory over alternative statistical methods in margin setting. The weaknesses of the other approaches including the underestimation of Gaussian estimates in extreme price movement modeling, the inability to deal with relatively low probability levels using the historical distribution and the over reliance on a particular period of time associated with conditional estimation are all documented.

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Appendix

Estimation of the shape parameter

This appendix describes the semi-parametric estimation procedure for the shape parameter of the extreme value distribution.

The widely used Hill (1975) moment estimator is used to determine tail quantiles and probabilities. The Hill estimator represents a maximum likelihood estimator of the tail index, the inverse of the shape parameter:

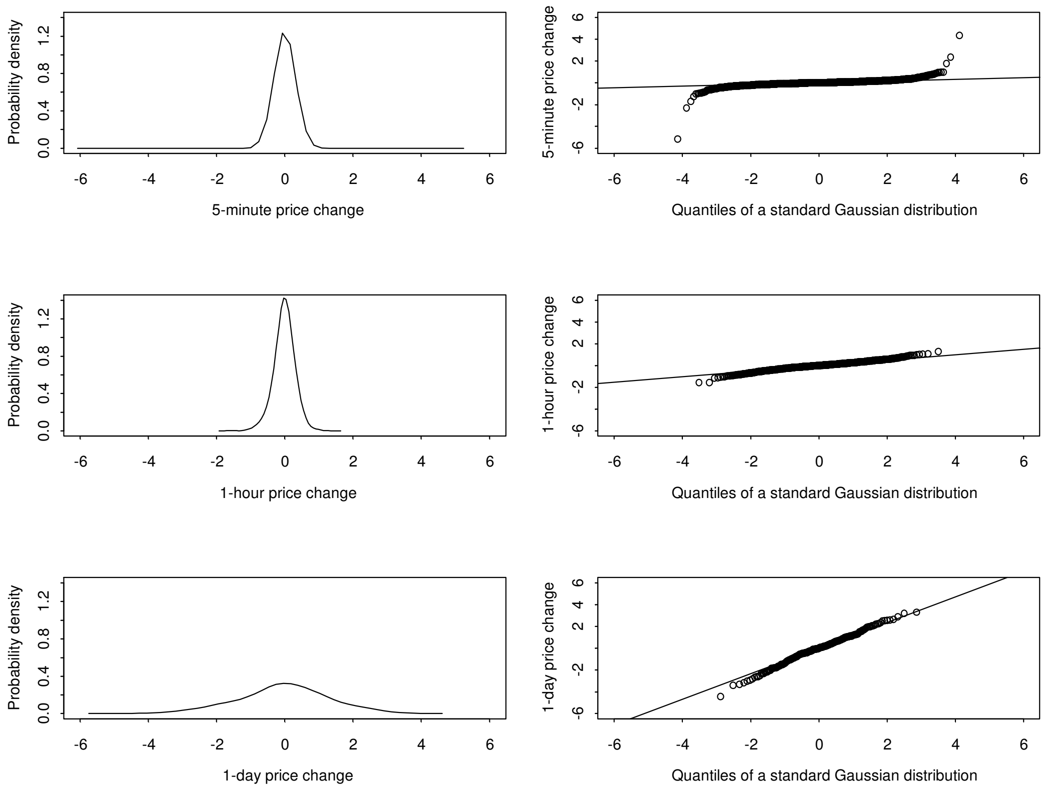
(A1) $\gamma = 1/\alpha = (1/m) \sum [\log r_{(n+1-i)} - \log r_{(n-m)}]$ for i = 1....m

focusing on the maximum upper order statistics. This tail estimator is asymptotically normal (de Haan *et al* (1994)):

(8) $(m)^{1/2}/(r_{m+1} \log(m/np))(r_p - E\{ r_p \}) \approx N(0, \gamma^2)$

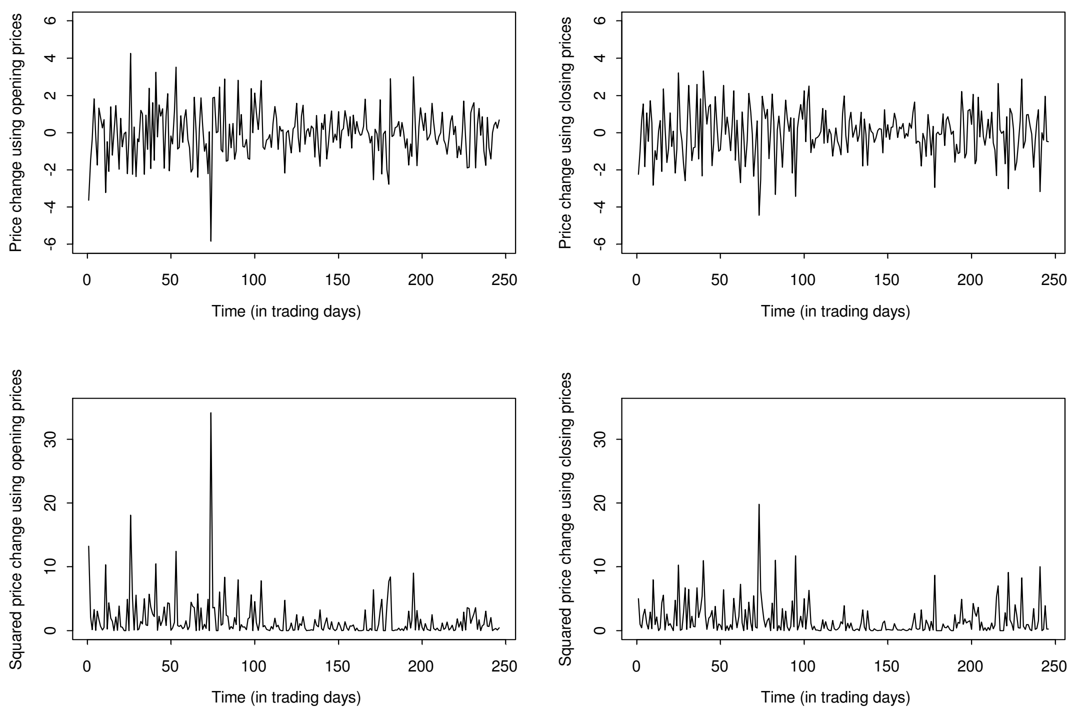
An estimation issue is determining the optimal number of tail values, m (see Danielson *et al* (2001) for a discussion). The dilemma faced is that there is a trade-off between the bias and variance of the estimator with the bias decreasing and variance increasing with the number of values used. The approach introduced by Huisman *et al* (2001) is applied here that performs well under simulation. The use of the Hill estimator in the literature is due to a number of factors. The estimator is the most widely used with the most desirable time series properties (Hall and Welsh (1984)) with specific support for its application to financial time-series from simulation studies of it versus other estimators based on order statistics (Kearns and Pagan (1997)). Also, the Hill estimator does not require the existence of a fourth moment, a characteristic that is strongly debated for financial data. Most importantly, the Hill estimator is the intrinsic part of a larger procedure used in this study that examines tail behavior. In fact, Dacarogna *et al* (1995) show that by applying the highest frequency data possible ensures that the shape parameter provides the most efficient estimator of tail behavior exploiting the fractal nature of extremes. Intuitively a large (high) frequency data set has more observable extremes that a small (low) frequency one over the same time interval thereby allowing for stronger inferences of these rare events. Thus estimation of relatively low frequency margins is best achieved by estimating shape parameter values at high-frequencies and using a simple scaling law to extend for these aggregated price changes. A simple scaling factor similar to the $\sqrt{n}$ used for normal distribution is applicable. The high-frequency margin estimates are adjusted by an α-root scaling law scaling ($k^{1/\alpha}$) with no additional estimation of extra parameters required.

Figure 1. Probability density function and QQ plot for price changes of the FTSE 100 contract.



Note: these figures represent the probability density function and the QQ plots for price changes in the FTSE 100 future contract for the year 2000. Three different frequencies are used to compute the price changes: 5 minutes, 1 hour and 1 day.

Figure 2. Daily price changes and daily squared price changes of the FTSE 100 contract.



Note: these figures represent the history of the price change and squared price change of the FTSE 100 future contract for the year 2000. Daily price changes are computed in two ways: from 9 am to 9 am on the following day (opening prices) and from 5 pm to 5 pm (closing prices).

Table 1. Basic statistics for the FTSE 100 contract price changes defined for different frequencies.
Panel A. Price changes

	Frequency of price changes		
	5-minutes	1-hour	1-day
Mean	0.00	-0.02	-0.03
Standard deviation	0.11	0.30	1.30
Skewness	-1.44	-0.28	-0.15
Kurtosis	254.5	1.54	0.26
Kolmogorov-Smirnov	0.08	0.05	0.04
test of normality	(0.00)	(0.00)	(0.31)
Minimum	-5.17	-1.57	-4.38
1st quartile	-0.05	-0.18	-0.77
2nd quartile	0.00	-0.00	-0.03
3rd quartile	0.05	0.16	0.76
Maximum	4.34	1.29	3.20

Panel B: Squared price changes

	Frequency of price changes		
	5-minutes	1-hour	1-day
Mean	0.01	0.09	1.70
Standard deviation	0.21	0.17	2.55
Skewness	107.99	5.24	2.69
Kurtosis	12 815.78	46.5	10.38
Kolmogorov-Smirnov	0.47	0.29	0.25
test of normality	(0.00)	(0.00)	(0.00)
Minimum	0.00	0.00	0.00
1st quartile	0.00	0.01	0.14
2nd quartile	0.00	0.03	0.65
3rd quartile	0.01	0.09	2.21
Maximum	0.01	0.10	19.17

Note: this table gives the basic statistics and empirical quantiles for the price changes (Panel A) and the squared price changes (Panel B). It also presents the results of the Kolmogorov-Smirnov test for normality with the *p*-value below in parentheses. Three different frequencies are used to compute the price changes: 5 minutes, 1 hour and 1 day. Data are price changes of the FTSE 100 future contract over the year 2000.

Table 2. Basic statistics for the FTSE 100 contract price changes defined with different time-intervals.

Panel A. Price changes

	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
Mean	-0.04	-0.04	-0.03	-0.03	-0.03	-0.03	-0.04	-0.03	-0.03
Standard deviation	1.32	1.23	1.20	1.23	1.18	1.29	1.22	1.16	1.30
Skewness	-0.13	-0.10	-0.30	-0.47	-0.32	-0.13	-0.14	-0.09	-0.15
Kurtosis	1.52	1.13	0.88	1.39	0.16	0.14	-0.05	-0.32	0.26
Kolmogorov-Smirnov	0.05	0.04	0.05	0.06	0.04	0.03	0.04	0.03	0.04
test of normality	(0.10)	(0.48)	(0.11)	(0.11)	(0.46)	(0.62)	(0.57)	(0.71)	(0.31)
Minimum	-5.84	-4.92	-4.74	-5.73	-4.48	-4.54	-3.60	-3.13	-4.38
1st quartile	-0.79	-0.86	-0.78	-0.76	-0.80	-0.79	-0.79	-0.80	-0.77
2nd quartile	-0.04	-0.01	0.02	-0.01	0.03	-0.02	-0.04	0.02	0.00
3rd quartile	0.78	0.74	0.73	0.81	0.80	0.86	0.78	0.76	0.76
Maximum	4.26	4.06	3.59	3.09	2.59	3.20	3.02	2.48	3.20

Panel B: Squared price changes

	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
Mean	1.73	1.51	1.44	1.51	1.40	1.65	1.48	1.35	1.70
Standard deviation	3.24	2.66	2.44	2.79	2.06	2.42	2.06	1.74	2.55
Skewness	5.38	4.49	4.27	6.58	4.03	3.15	2.25	1.75	2.69
Kurtosis	43.77	27.90	26.24	65.77	27.84	16.08	5.94	2.90	10.38
Kolmogorov-Smirnov	0.30	0.29	0.28	0.29	0.25	0.25	0.24	0.22	0.25
test of normality	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1st quartile	0.09	0.13	0.13	0.14	0.13	0.18	0.16	0.12	0.14
2nd quartile	0.63	0.62	0.58	0.60	0.64	0.72	0.63	0.60	0.58
3rd quartile	2.05	1.70	1.74	1.94	1.85	1.86	1.95	1.76	2.21
Maximum	34.13	24.24	22.46	32.87	20.06	20.60	12.93	9.79	19.17

Note: this table gives the basic statistics and empirical quantiles for the price changes (Panel A) and the squared price changes (Panel B) over different time-intervals. It also presents the results of the Kolmogorov-Smirnov test for normality with the *p*-value below in parentheses. To define the price change, the starting time, which is equal to the ending time on the following day, varies from 9 am (opening of the market) to 5 pm (closing of the market). Data are price changes of the FTSE 100 future contract over the year 2000.

Table 3. Shape parameter estimates and test of the existence of moments.

Panel A. Left tail

	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
Shape parameter α	3.06 (0.65)	3.25 (0.69)	2.68 (0.57)	3.30 (0.70)	3.62 (0.77)	3.51 (0.75)	6.34 (1.35)	3.03 (0.65)	3.11 (0.66)
H0: α>2	1.63 (0.45)	1.81 (0.46)	1.18 (0.38)	1.85 (0.47)	2.10 (0.48)	2.02 (0.48)	3.21 (0.50)	1.60 (0.45)	1.68 (0.45)
H0: α>4	-1.43 (0.00)	-1.08 (0.00)	-2.32 (0.00)	-0.99 (0.00)	-0.49 (0.00)	-0.65 (0.00)	1.73 (0.46)	-1.50 (0.00)	-1.33 (0.00)

Panel B. Right tail

	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
Shape parameter α	2.58 (0.55)	3.63 (0.77)	4.34 (0.93)	3.77 (0.80)	4.20 (0.90)	3.48 (0.74)	4.96 (1.06)	4.08 (0.87)	3.64 (0.78)
H0: α>2	1.05 (0.35)	2.11 (0.48)	2.53 (0.49)	2.20 (0.49)	2.46 (0.49)	2.00 (0.48)	2.80 (0.50)	2.39 (0.49)	2.11 (0.49)
H0: α>4	-2.59 (0.00)	-0.48 (0.00)	0.37 (0.14)	-0.29 (0.00)	0.22 (0.09)	-0.70 (0.00)	0.91 (0.32)	0.09 (0.04)	-0.47 (0.00)

Note : this table gives the shape parameter estimates for the left tail (Panel A) and the right tail (Panel B) of the distribution of daily price changes and a test of the existence of the moments of the distribution. The first line of the table gives the shape parameter estimate obtained with the method developed by Huisman *et al* (2001) with the standard error below in parentheses. The second and third lines give the results of a test of the existence of the second moment (the variance) and the fourth moment (the kurtosis) with the *p*-value below in parentheses. As the shape parameter corresponds to the highest moment defined for the distribution, the null hypotheses are defined as follows: H_0: α>2 and H_0: α>4. To define the price change, the starting time (which is equal to the ending time on the following day) varies from 9 am (opening of the market) to 5 pm (closing of the market). Data are price changes of the FTSE 100 future contract over the year 2000.

Table 4. Margin levels for given probabilities based on daily price changes.

Panel A. Long position

Probability (waiting period)	Model	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
95% (20 days)	Gaussian	2.21	2.06	2.00	2.05	1.97	2.15	2.05	1.94	2.17
	Extreme value	1.85	1.95	1.89	1.84	2.04	1.83	1.85	1.95	2.05
	Historical	1.90	1.87	2.23	2.08	2.34	2.14	2.04	2.28	2.28
	APARCH	2.05	2.22	2.63	2.55	3.19	2.94	2.65	2.90	2.91
99% (100 days)	Gaussian	3.11	2.90	2.82	2.89	2.78	3.03	2.88	2.73	3.05
	Extreme value	2.94	3.22	3.12	2.70	2.78	2.42	2.26	2.74	2.93
	Historical	2.98	3.23	3.06	2.76	2.90	2.89	2.51	3.19	3.25
	APARCH	3.62	3.62	3.62	3.12	3.90	3.85	3.29	4.38	4.39
99.60% (250 days)	Gaussian	3.54	3.30	3.21	3.29	3.16	3.45	3.28	3.11	3.48
	Extreme value	3.83	4.29	4.15	3.35	3.32	2.84	2.54	3.32	3.59
	Historical	3.59	3.39	3.41	3.01	3.01	3.10	2.71	3.31	3.45
	APARCH	4.13	3.92	3.85	3.73	4.88	4.02	3.55	4.77	4.67
99.80% (500 days)	Gaussian	3.84	3.58	3.48	3.57	3.43	3.74	3.55	3.37	3.77
	Extreme value	4.67	5.32	5.15	3.95	3.79	3.20	2.77	3.84	4.18
	Historical	na	na	na	na	na	na	na	na	na
	APARCH	4.88	4.61	6.51	4.99	6.51	4.91	3.63	5.51	5.43

Panel B. Short position

Probability (waiting period)	Model	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
95% (20 days)	Gaussian	2.13	1.98	1.94	1.99	1.91	2.09	1.97	1.88	2.11
	Extreme value	1.70	1.76	1.80	1.65	1.96	1.74	1.77	2.06	1.94
	Historical	1.85	1.72	1.75	1.73	2.06	2.03	1.92	2.19	2.10
	APARCH	2.28	2.14	2.33	2.16	3.12	2.86	2.66	3.33	3.24
99% (100 days)	Gaussian	3.03	2.82	2.76	2.83	2.72	2.97	2.80	2.67	2.99
	Extreme value	2.69	2.91	2.98	2.41	2.67	2.31	2.16	2.89	2.77
	Historical	2.76	2.82	2.67	2.47	2.82	2.50	2.37	2.78	2.77
	APARCH	3.51	3.38	3.68	3.13	5.22	3.97	3.33	4.51	4.51
99.60% (250 days)	Gaussian	3.42	3.46	3.22	3.15	3.23	3.10	3.39	3.20	3.05
	Extreme value	3.87	3.87	3.97	2.99	3.18	2.71	2.42	3.51	3.40
	Historical	3.70	3.01	2.90	2.58	2.97	2.70	2.48	2.96	3.20
	APARCH	4.50	4.45	3.83	3.22	5.56	4.55	3.47	4.93	5.40
99.80% (500 days)	Gaussian	3.76	3.50	3.42	3.51	3.37	3.68	3.47	3.31	3.71
	Extreme value	4.80	4.80	4.93	3.53	3.63	3.05	2.63	4.06	3.96
	Historical	na	na	na	na	na	na	na	na	na
	APARCH	4.94	4.55	3.96	3.25	5.87	4.60	3.54	5.14	5.76

Note : this table gives the margin level for a long position (Panel A) and a short position (Panel B) for different probability levels ranging from 95% to 99.8% or equivalently different waiting periods ranging from 20 trading days (1 month) to 500 trading days (2 years). Different statistical models are used: three unconditional distributions (the Gaussian distribution, the extreme value distribution and the historical distribution) and a conditional process (the Asymmetric Power ARCH or APARCH). The historical estimates are not available (na) for out of sample inferences due to data unavailability. To define the price change, the starting time (which is equal to the ending time on the following day) varies from 9 am (opening of the market) to 5 pm (closing of the market). Data are price changes of the FTSE 100 future contract over the year 2000.

Table 5. Extreme value probabilities for given margin levels.

Panel A. Long position

Margin level	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
-5%	0.39	0.60	0.54	0.18	0.12	0.04	0.00	0.14	0.22
	(2.57)	(1.66)	(1.84)	(5.51)	(8.60)	(26.29)	(237.08)	(7.03)	(4.53)
-10%	0.03	0.06	0.06	0.01	0.00	0.00	0.00	0.01	0.01
	(28.73)	(15.39)	(16.90)	(102.89)	(318.81)	(1418.71	(62485.51)	(187.70)	(103.83)

Panel B. Short position

Margin level	Open	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	Close
-5%	0.29	0.43	0.47	0.11	0.09	0.03	0.00	0.19	0.17
	(3.49)	(2.30)	(2.11)	(8.84)	(10.68)	(34.60)	(349.89)	(5.40)	(5.81)
-10%	0.03	0.05	0.05	0.01	0.00	0.00	0.00	0.01	0.01
	(38.96)	(21.34)	(19.44)	(165.15)	(395.93)	(1867.17)	(92216.90)	(144.21)	(133.13)

Note : this table gives the extreme value distribution probability levels and the corresponding waiting periods below in parentheses for given margin levels for a long position (Panel A) and a short position (Panel B). Two margin levels are considered: ±5% and ±10%. To define the price change, the starting time (which is equal to the ending time on the following day) varies from 9 am (opening of the market) to 5 pm (closing of the market). Data are price changes of the FTSE 100 future contract over the year 2000.

Table 6: Daily margin levels obtained with the extreme value distribution based on 5-minute, 1-hour and 1-day price changes.

Panel A. Long position

Probability (waiting period)	Frequency of price changes		
	5 minutes	1 hour	1 day
95% (20 days)	1.87	1.92	1.92
99% (100 days)	3.09	2.91	2.79
99.60% (250 days)	3.34	3.68	3.47
99.8% (500 days)	4.05	4.39	4.10

Panel B. Short position

Probability (waiting period)	Frequency of price changes		
	5 minutes	1 hour	1 day
95% (20 days)	1.81	1.54	1.82
99% (100 days)	3.03	2.41	2.64
99.60% (250 days)	3.12	2.99	3.32
99.8% (500 days)	3.78	3.53	3.93

Note : this table gives the daily margin levels obtained with the extreme value distribution for a long position (Panel A) and a short position (Panel B) for different probability levels ranging from 95% to 99.8% or equivalently different waiting periods ranging from 20 trading days (1 month) to 500 trading days (2 years). Three different frequencies are used to compute the price changes: 5 minutes, 1 hour and 1 day. Margin levels obtained with 5-minute price changes and 1-hour price changes are scaled to obtained daily margin levels. Margin levels obtained from daily price changes correspond to the average over the margin levels obtained with different time-intervals. Data are price changes of the FTSE 100 future contract over the year 2000.