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The Non-Linear Evolution of High Frequency Short  
Term Interest Rates

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University College Dublin

2<sup>nd</sup> April 2008

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# **The Non-Linear Evolution of High Frequency Short Term Interest Rates**

## **Abstract**

In this paper new results are documented regarding the short term evolution of global short term interest rates. Much work has been carried out concerning the evolution of interest rates over long time scales, on the order of one month or greater. However high frequency data has only been considered in a limited number of studies. In this study the evolution of the short term yield curve, on a day to day basis, is considered and results are presented that suggest that over these short time scales, short term interest rates exhibit non-linear autoregressive behaviour, in contradiction of the efficient markets hypothesis. In addition the high frequency data indicates that the observed co-movement across currencies of longer maturity interest rates result from a vector error correction process (VECM). Greater information on the nature of the process may be obtained by considering a non-linear VECM process. Based on the output of both non-linear uni-variate and multi-variate models, limited short term statistically significant predictions of the evolution of various short term interest rate instruments may be carried out.

**Key Words:** interest rates, non-linear, high frequency

**J.E.L. Classification** C32, C33, E47

## **1: Introduction**

Yield curve modelling, from its very inception was based on the concept that the yield curve formed part of an overall economic model and could thus be described in terms of economic variables (Cox, Ingersoll et al. 1985). Significant progress has been made more recently with the introduction of the New Keynesian Phillips Curve approach (Clarida, Gali et al. 1999). This has allowed for the development of yield curve models where economic variables such as inflation and output, and their expectations are explicitly utilised. In addition, considerable effort has been put into attempting to understand the dynamic evolution of interest rate yield curves in terms of dynamic macroeconomic and other information flows (Piazzesi 2001). This work has been motivated by a number of concerns. Firstly, it has been shown that the shape of the yield curve may be a good predictor of economic recession, up to twelve months in the future (Estrella and Hardouvelis 1991). More recent work has looked at the evidence of a relationship between yield curves and actual economic variables (Estrella 2005), (Ang, Piazzesi et al. 2006). In addition considerable analysis has been done using large scale vector autogressions involving market and macroeconomic data (Ang and Piazzesi 2003).

In addition to the large scale, long range studies mentioned above, some work has been carried out that looks at the impact of surprise news events on the evolution of the yield curve. Initial work concentrated on the impact of economic data (Ederington and Lee 1993) however the study was limited because it was not possible to control for expectations on the data, which would, implicitly, be already be represented in the yield curve data. Economic forecast survey may give an indication of expectations of economic data and thus may be used properly characterise the “surprise” element of economic news data. Taking this approach has shown that the yield does evolve, over the short term, in a manner broadly consistent with economic theory, upon the release of economic surprise data (Gürkaynak 2005).

However it would seem important that prior to carrying out this type of analysis, the yield curve data is considered on an independent basis. This is to ensure

that its behaviour has been thoroughly analysed so that when macroeconomic data is added to the analysis its impact may be properly determined (Balduzzi, Elton et al. 2001). This is the motivation behind the current study. We seek to characterise, in a statistical manner, the daily evolution of short term interest rates so that later studies, on the impact of macroeconomic data may be carried out in a proper context. If the short term interest rate market is truly efficient, then it would be expected that no structure would be observed, outside of those effects due to auto-correlated news surprises.

The paper is laid out as follows. Section 2 considers the data that is used in the analysis, how it is collected, and the methodology that is used to “clean” it before the analysis is carried out. Section 3 deals with the analysis methodology and gives a heuristic motivation for the use of the different non-linear autoregressive models, both univariate and multi-variate. Section 4 describes the different metrics that will be used to analyse the results of applying the models. Section 5 describes the results when the models are applied to different markets and products. Section 6 is concerned with forecasts that may be made using the models. Section 7 advances a heuristic argument that attempts to explain the observed phenomena in terms of asymmetric utility. Section 8 concludes the paper.

## **2: Data**

The short term interest rate data used in this study was collected by the British Bankers Association (BBA). These are short term interbank deposit rates (LIBOR – London Interbank Offered Rate), with maturities ranging from 1 month to 12 months. The currencies that will be considered are US dollar (USD), Euro (EUR), Swiss Franc (CHF), and United Kingdom Pound (GBP). In addition, in order to extend the analysis to expectation based instruments, 3 month EURIBOR short term interest rate futures contract data will be considered.

The LIBOR data is freely available on the web and is provided by the British Bankers Association (BBA). The data that is saved by the BBA is collected on a daily

basis using a polling methodology. This ensures a high level of confidence in the accuracy of the data, reflected in the fact that swap/cap/floor contracts are settled against these quoted rates. In situations where an insufficient number of banks (normally 5) actually reply to the request for data, the value for that day is left blank. Whilst this does have the result that the time series data is not effectively available for every day in the sample, it does mean that every time point that is available is a valid point and not some artefact of an ex-post interpolation routine. A more detailed description of the sampling methodology may be found on the website of the BBA.

The short term interest rate futures contract data is the closing data for the 3 month EURIBOR contract that is traded on the Eurex exchange. This closing data is supplied by the exchange for contracts of a range of maturities extending up to 5 years from the expiry date of the closest to maturity contract.

It is important to note that in analysing high frequency data, asynchronicity in the data collection times, that is often ignored for longer time scales, such as monthly data, may not be so easily disregarded. As a result, vector relationships between EUR, USD and JPY data are not considered in this study.

The time period under consideration in this study is from January 2000 to March 2007. The start of the period is determined by availability of liquid data. The end point is chosen somewhat arbitrarily, however specifically to avoid using data from the latter half of 2007 when credit and liquidity issues were as relevant to the setting of short and long term interest rates as expectations of future interest rates and inflation.

### **3: Non Linear Autoregressive Models**

It has been shown that the current level of an interest rate is a better predictor of its future value than the no-arbitrage calculated forward rate which in foreign exchange forward rates is known as the “forward premium puzzle” (Fama 1984). From this and from the efficient markets hypothesis, it has been assumed that the current prices of market instruments contain all the information relevant to their

pricing. Thus one would expect that the change in the interest rate on a day to day basis is effectively independent of the previous day's rate, or indeed the previous day's change of rate from the day before. In other words there is an assumption that interest rate dynamics follow a Markov type process. This assumption is easily tested on the data using an Adjusted Dickey-Fuller (ADF) test (Dickey and Fuller 1979).

From the above a working hypothesis for the temporal evolution of interest rate markets is that they are effectively determined by the introduction and assimilation of new information into that market (Green 2004). Assuming that all of this information is assimilated in a very short timescale, less than one day, there should be no information in previous day's rates that would influence the change of rates on subsequent days, outside of the possibility that the information flow itself is correlated. If on the other hand, any new information is not properly assimilated on its day of release then it is possible that there is movement on subsequent days that is due to the original inappropriate response (either an over- or an under-reaction). This may mean that differences on subsequent days may in fact have a relationship to the difference in rates experienced on the day of release of the relevant information. Given the large number of possible events that may influence market behaviour, it is not assumed that there is only a single piece of information that may or may not be properly assimilated on any given day.

From an accounting point of view, if the market does not properly assimilate the information that has been released to it, there are only two possible inferences to be made. Either the market under-reacted to the new information or it overreacted to the new information. The third possibility that it's reaction was appropriate, but for the wrong reasons is somewhat otiose. It may be possible to observe such behaviour by comparing the differences in market levels on subsequent days and see if there is a statistically relevant relationship between the differences.

As noted above, if there is little or no reaction to information flow, then the time series of the market rate, on its own, should not be useful in determining future value of the rate. As well as this, if there is little new or relevant information on a given day, then the differences in the rate on that day should not provide any real information of the differences in subsequent days. A third reason for changes in a

given interest rate are the activities of the so-called noise traders who have been the subject of many papers in behavioural finance (Kyle 1985). Whilst it is not the purpose of this paper to analyse the activities of such traders, it is necessary to reduce their influence on the analysis. These criteria effectively point towards the use of non-linear models, and in particular threshold models for the analysis of the time series data. Whilst the use of threshold models may cause significant amounts of potentially useful data to be thrown away, at the same time, it is a very effective filter to ensure that only times where there has been significant level of cumulative information flow and/or assimilation on a given day are considered.

Threshold models have been considered at length in econometric literature (Tong 1990) and more recently have more recently been applied in numerous different ways to interest rate data (Gospodinov 2005). This and other papers concentrate on econometric impacts or stochastic yield curve models.

The purpose of this study is somewhat different and considerably less ambitious. The data that will be considered is daily interest rate data which is assumed to move under the influence of new information and the expectation of new information. It is considered highly unlikely that responses of the markets to both new information and that information's expectation will be constant over the six years of the sample data. In addition, it is somewhat unlikely that the response to different types of information at different times will be similar or even linear over time. As such, the exact specification all the time series models considered will be kept as simple and as general as possible. In addition the diagnostic tests of the models will attempt to encompass as broad a spectrum of results as possible.

The initial model that will be considered here is a TAR(1) model in the differences of any given interest rate time history (Campbell and Shiller 1991).

First the times series of the first differences is created for any instrument "i". It should be noted that this can vary across both currency and maturity:



$$x_{i,t} = r_{i,t} - r_{i,t-1}$$

Here  $r_{i,t}$  is the value of the  $i^{\text{th}}$  currency rate at time  $t$ . The TAR(1) model that is to be considered is the following format:

$$\begin{aligned} x_{i,t} &= \varepsilon_t & \forall |x_{t-1}| \leq \tau \\ x_{i,t} &= \alpha * x_{i,t-1} + \varepsilon_t & \forall |x_{t-1}| > \tau \end{aligned}$$

Where  $\varepsilon_t$  is a random variable whose variance is the square of the local basis point volatility of the interest rate. As the data has not been de-trended, it is clear that over the whole data set the expectation is not exactly zero, but this will be very small compared with the volatility of the underlying variable. Thus this model may be thought of as the time series being effectively a martingale if the previous day's value is less than a given level, and being autoregressive if greater than that level. In other words the past has an impact, only if it is of large enough absolute magnitude. The influence of the past is a proportional to the magnitude of the previous level. However given that a threshold (non-linearity) is being used in the specification of the model, it may not be appropriate to have a linear reaction to the move within the autoregression expression.

The concept of a non-linear response in a non-linear model has been looked at from a large number of avenues, generally involving a marked increase in the number of parameters that need to be fitted. Given the lack of stationarity of the data, it seems appropriate that the principle of parsimony should be applied strenuously. Thus a second type of threshold model is proposed where the autoregressive component is determined by the sign of the previous day's value. This threshold sign regression - TSR(1) model is characterised by the following

$$x_{i,t} = \varepsilon_t \quad \forall |x_{t-1}| \leq \tau$$

$$x_{i,t} = \beta * \text{sign}(x_{t-1}) + \varepsilon_t \quad \forall |x_{t-1}| > \tau$$

This model is characterised by the hypothesis that if the change in the previous day's value is sufficient to exceed the threshold, this has an impact that is related to that fact, not to the magnitude of the change in the level of the rate. This model in its non-threshold form was considered in earlier work (Granger and Terasvirta 1999) where it was shown to exhibit similar properties to a fractionally integrated model. These two models may obviously be combined to produce the third type of uni-variate model to be considered in this paper - the Threshold Auto and Sign Regression model – TASR(1,1)

$$x_{i,t} = \varepsilon_t \quad \forall |x_{t-1}| \leq \tau$$

$$x_{i,t} = \alpha * x_{t-1} + \beta * \text{sign}(x_{t-1}) + \varepsilon_t \quad \forall |x_{t-1}| > \tau$$

It may be noted that if there is a statistically significant threshold response to preceding data exceeding the threshold, the signs of  $\alpha$  and  $\beta$  will be opposite.

In applying the models to the data it is important that the data is not over-fitted. In other words it is necessary that there are sufficient threshold events to ensure that there is an appropriate level of statistical relevance to the two data sets – those events above and those events below the threshold. Typically in the relevant literature the ratio of the number of threshold events, divided by the population size -  $\pi^*$  must be greater than 0.15 for the sample to be considered.

It has been shown in numerous studies that individual yield curves evolve with reference to themselves across a wide range of environments (Dai and Singleton 2003). In other words, yield curves tend to evolve as a single integrated unit rather than as individual components that are related in only a statistical manner. In addition,

if the yield curve may be considered to be determined by the expectation of future interest rates, then it may be expected that the expectation of rates at a given date in the future will impact significantly the expectation of rates at subsequent maturities. This observation is the basis for constructing yield curve models that seek to understand the evolution of the yield curve in terms of a limited number of observed or unobserved parameters. Typically however such time series analysis has normally been carried out over long time scales, such as monthly or quarterly and has been linked to asynchronous economic data (Campbell and Ammer 1993). In this section a simple multiple time series analytic framework is described that will be used to study the high frequency evolution of different yield curve instruments such as short term interest rate futures and interest rate swaps. In other words the univariate analysis allows for the examination of interest rate data on an individual basis. The vector analysis will identify whether or not there are statistically significant relationships between short term interest rates of different maturity and/or currency.

The motivation for using a vector type framework to describe the co-evolution of interest rates is based on the expectations hypothesis of interest rates. That is for two interest rate instruments of different maturities, the information on the expectation of future interest rates in the level of the shorter maturity instrument, will also be included in the determination of the longer maturity instrument. Thus changes in the level of the shorter maturity instrument must be part of the evolution of the longer maturity instrument. It may also be possible for longer maturity instruments to impact the evolution of short maturity ones if the longer maturity instrument is more liquid. In addition, significant co-movement has been observed over longer time-scales (monthly) in a number of studies (Driessen, Melenberg et al. 2003). As such, a vector framework is appropriate for attempting to identify such behaviour in high frequency data, if it exists.

The standard multiple time series framework is a vector autoregression. However, in considering short term interest rate data a number of stylised facts need to be considered. Firstly, numerous studies have identified interest rate data as being co-integrated of order approximately one. Secondly, yield curves modelling is generally carried out using some type of bootstrap methodology (James J 2000) and

as such are constructed from short maturities to longer maturities. These “facts” point toward the use of a vector error correction methodology (VECM) and this is what is considered in this case. A complete review of such models is given in (Lutkepohl 2005). A two factor VECM is specified by the following

$$\Delta x_t = \alpha_1(x_t - \beta \cdot y_t) + \gamma_{11} \cdot \Delta x_{t-1} + \gamma_{12} \cdot \Delta y_{t-1} + \eta_t^x$$

$$\Delta y_t = \alpha_2(x_t - \beta \cdot y_t) + \gamma_{21} \cdot \Delta x_{t-1} + \gamma_{22} \cdot \Delta y_{t-1} + \eta_t^y$$

Where  $\Delta x_t = (x_t - x_{t-1})$  and  $\eta$  are the innovation terms for the autoregression.. Using the VECM, allows us to build on the precious univariate methodology to investigate if high frequency changes in yields of different maturity may have some further structure that is related to the differential absorption of surprise information.

These equations may be generalised using matrix methodology to yield

$$\Delta X_t = \Pi \cdot X_t + \Gamma \cdot \Delta X_{t-1} + U$$

where  $X$  is a  $k$  dimensional time series matrix and where  $\Pi$  and  $\Gamma$  are matrices. A nonlinear generalisation of this approach may be found by altering the form of the VECM based on the relative or absolute magnitudes of the components of  $\Delta X_{t-1}$ .

#### **4: Statistical Diagnostics**

Given the lack of continuity in the actual datasets and the non-linearity in the proposed analysis methods, it is difficult to use a large number of the traditional basic econometric methods that would be used to test statistical significance in the analysis of a time series. As such whilst they may be applied mechanically, it is not clear whether the results of tests based on autocorrelation, Dickey Fuller, Box-Ljung etc are reliable given the effective non-linearity in time of the data set used. This is primarily

due to the discontinuities in the time series, due either to the data sampling methodology or the threshold characteristics of the models.

With respect to the uni-variate analysis Tong has shown the least squares may be used to fit threshold models and appropriate statistical errors of these fitted values may be determined in the normal fashion. Illustrating that that fitted values of the models are significantly different from zero will be the first stage in determining whether or not the model that is being tested has any validity.

In the estimation of the VECM, both generalised least squares (GLS) or maximum likelihood (ML) techniques may be used. Given the large number parameters that are to be estimated, great care must be taken to ensure the reliability of all of the estimated parameters. In simulations designed to test the models, GLS and ML methods gave very similar parameter estimations and standard errors for data sets in excess of 100 datapoints.

In order to test the validity of the different models in a consistent manner, Tong suggests the use of a modified Aikake Information Criterion (AIC). However, given the large number of parameters to be estimated in the VECM, which could lead to over-fitting issues, it would seem appropriate to consider an information criterion that heavily penalises the number of parameters. Consistent with this, a modified Bayesian Information Criterion (m-BIC) will be considered in order to assess the fit quality (Schwarz 1978). Minimising the modified BIC may be used to determine the most appropriate model, linear or non-linear to specify the time series of data.

The modified BIC may be estimated by the following:

$$BIC(p_1, p_2) = n_1 \cdot \log(ssr_1) + n_2 \cdot \log(ssr_2) + (p_1 + 1) \log(n_1) + (p_2 + 1) \log(n_2)$$

Where  $n_j$ ,  $j = 1, 2$  is the number of observations in the  $j^{\text{th}}$  regime,  $ssr_j$ ,  $j = 1, 2$  is the sum of the square of the residuals in the  $j^{\text{th}}$  regime. The  $p_j$ ,  $j = 1, 2$  are the number of variable to be determined in each regime. For the models that are to be considered,  $p_1 = 0$  and  $p_2 = 1$  for the TAR and TSR models and  $p_2 = 2$  for the TSAR model. For the full VECM,  $p_1 = 0$  and  $p_2 = 8$ . In addition, in fitting the VECM, it is possible to reduce the m-BIC by omitting parameters that do not contribute to the overall fit.

In comparing the m-BIC of the non-linear model with that of the null hypothesis of zero autoregression care must be taken to determine the m-BIC of the unfitted data for each value of the threshold. This merely reinforces the point that the sum of the m-BIC's is not equal to the m-BIC of the sum.

The previous two tests refer to the appropriateness of a given model to be used in describing the time series. They will generally be used to determine whether or not one or indeed all of the previously described models give a better description of the relevant time series than the null hypothesis – a martingale type of motion. The following two tests will be used to test the predictions of the models when they are applied out of sample.

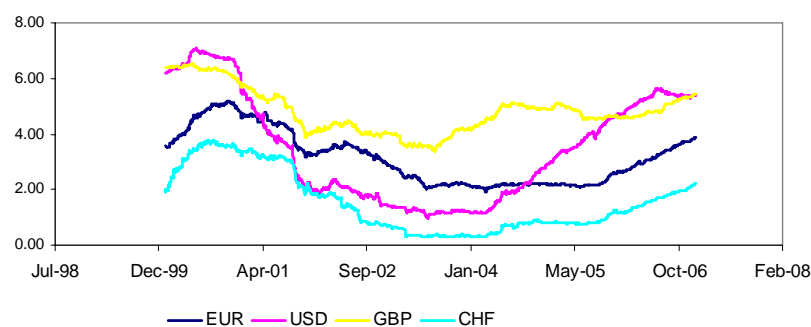
Firstly, it is proposed to use a Diebold-Mariano test (Diebold and Mariano 1995). This effectively compares the error functions of the predictions of different models and constructs a statistic to determine whether or not the predictions of one model are statistically superior to the other. Results shown in the original paper illustrate that this is in fact a very useful test in practice. This test effectively gives a statistical estimate of the real accuracy of the predictions of the model.

It is clear however that in making predictions in the financial markets, whilst it would be favourable if one could predict both the sign and magnitude of the next days movement, it would undoubtedly be useful if one could predict just the sign of the next day's movement, under the assumption that the losses and gains are similar for both accurate and inaccurate predictions. In addition, if one is looking to understand the co-movement of interest rates, the fact that they may be moving, in the same direction, under a non-linear schema is of value in attempting to understand the causes of the co-movement. In order to test the accuracy of the sign of the prediction, the direction accuracy (DA) test (Pesaran and Timmermann 1992) may be used. This compares the sign of the predictions of the model with the signs of the realised results, and a test statistic is determined where the asymptotic standard normal distribution is obtained under the null hypothesis that the realised result and the prediction are independently distributed.

## 5: Results

In carrying out the analysis a range of currencies and maturities were considered. The currencies were EUR, USD, GBP and CHF. The maturities were 1mo, 3mo, 6mo and 12mo. In addition to this, for each currency-maturity pair, the appropriate threshold needs to be determined. Chan (Chan 1993) gives a methodology for determining a super-efficient estimate of the “best” threshold parameter, however at this point, given the distribution of the data and the lack of certainty in the actual true specification of the model, a range of threshold parameters for each pair are considered and the m-BIC’s for the different regimes are compared.

The following results will concentrate on those obtained from 6mo Libor data. As can be seen from Figure 1, the data sets at different times trend in different directions, or indeed display no trend at all. However over the period in question, 1999 – 2006, there is no significant gross movement in the level of short term interest rates across all the currencies considered.



*Figure 1: The time history of 6mo Libor rates for different currencies.*

In the Table 1 the in-sample results for different currencies are given using exclusively 6mo Libor rates for the different currencies. In analysing the data tau was varied from 0bp to 5bps in steps of 0.005bps. In the table below that data that was shown is that corresponding to the highest value of tau, consistent with  $\pi^* > 0.15$ .

As can be seen from Table 1, using a threshold model on the short rate data generally produces a reduced  $m - BIC$  for the data. In the cases of EUR and USD the TAR(1) was the “optimal model”. In both cases the value of  $\alpha$ , the threshold autoregressive parameter was significantly different from zero, indicating that the model does have real statistical significance. In the cases of GBP and CHF, the TSR(1) model was chosen. However in the CHF case the value of  $\beta$  is not significantly different from zero  $-0.0088 \pm 0.0064$ . As such, even though the BIC is reduced from that of the null hypothesis, it is not clear that it is appropriate to apply this model in attempting to describe the daily rate differences of CHF data. In the case of the GBP data, the value of  $\beta$  is significantly greater than zero, and this gives increased confidence that this model may provide some descriptive power for the movement of short term GBP interest rates.

*Table 1: Results from the 6 mo LIBOR short term interest rate for four currencies. The standard errors of the derived parameters are given in the rows below each parameter.*

Curr	$\tau$	Null BIC	TAR(1)		TSR(1)		TASR(1,1)		$\alpha$
			BIC	$\alpha$	BIC	$\beta$	BIC	$\beta$	
EUR	2	-1497	-1509	0.161	-1508	0.007	-1504	0.003	0.099
				0.003		0.003		0.067	0.003
GBP	1.5	-1738	-1745	0.109	-1753	0.006	-1748	0.006	-0.009
				0.002		0.002		0.050	0.002
USD	2	-861	-889	0.247	-876	0.011	-884	0.016	0.227
				0.012		0.009		0.061	0.004
CHF	2	-1151	-1153	0.125	-1159	0.009	-1154	0.009	-0.014

It is worth noting, that in the cases where the TSR model was chosen from the minimum BIC criteria, the parameters of the TASR model have the opposite sign. As noted earlier in the passage this indicated that in this case there is a significantly non-linear response to the threshold being exceeded in this area. In the case of GBP, the BIC of the TASR(1,) is less than that of the TAR(1), however the standard errors of



the  $\beta$  parameter mean that it cannot be considered to be significantly different from zero.

Using this insample data, the Diebold-Mariano and Directional Accuracy test may be performed to get an estimate of how accurately the insample data is predicted by the models. The results are given in Table 2. Given that both tests exhibit asymptotic normal  $N(0,1)$  behaviour a number of conclusions may be drawn from these results. Firstly, across the different currencies, the results of the Diebold-Mariano test are generally not statistically significant except for sterling. Thus from this test's point of view, it is not possible to distinguish between the results of the threshold models and the null hypothesis of random movement. However the results from the directional accuracy test are markedly different. In the currencies where the BIC and standard error test implied some validity to the application of the threshold model, the models provide a statistically significant prediction of the sign of the data on the following day.

*Table 2: The Diebold-Mariano Sign test and the Directional Accuracy test as applied to the insample data for the different currencies.*

Curr	Maty	Data	Events	SR	TAR(1)		TSR(1)	
					DM	DA	DM	DA
eur	6m	1590	257	61.1%	1.81	3.53	1.31	3.53
gbp	6m	1527	313	63.9%	3.67	4.81	1.53	4.81
usd	6m	1527	336	59.5%	-0.33	3.49	-1.20	3.49
chf	6m	1527	316	56.0%	1.35	2.12	0.68	2.12

The univariate analysis indicates the presence of significant non-linear activity in the high frequency evolution of short term interest rate data. Effectively the univariate results are a nested example of the multi-variate analysis. Aside from this special case, the multi-variate analysis may be applied to maturity panel data for individual currencies and across different currencies. As explained above due to the effects of asynchronous data sampling, the number of cross currency pairs is limited to EUR, GBP and CHF data. Two types of VECM will be considered. A first limited model where no autoregression is considered - NAR ( $\Gamma = 0$ ) and a second where a full 8 parameter VECM is studied. Two scenarios will be considered. Without using

any threshold, the two types of VECM are fitted to 1mo, 3mo, 6mo and 12mo EUR, USD, GBP and CHF LIBOR. For these datasets, the m-BIC's for the null case, and the two models are compared

As can be seen from table 3, the VECM produces the lowest m-BIC for combinations of very short maturity interest rates. However, except for the CHF data, no statistically significant VECM process is observed for longer maturities. The cross currency fits are shown in Table 4.

These results are broadly consistent across maturity where there is evidence of a VECM relationship between the EUR and GBP yield curves. Across all maturities against USD interest rates, the VECM has the lowest m-BIC. This is an example of the impact of asynchronous data sampling, where the 5 hour difference in sample times generates an effective, statistically significant vector autocorrelation assuming any non-zero co-movement of the interest rates. This however does not indicate if there is any real relationship between the dynamics of the interest rates of the two currencies

*Table 3: The m-BIC for three intra-currency models, null hypothesis (Null), non-autoregressive VECM (NAR) and the full VECM (VECM) using EUR, USD, GBP and CHF data. The minimum m -BIC is identified using a “\*”*

	Null	3mo NAR	VECM	Null	6mo NAR	VECM
EUR						
6mo	457.6	422.8	405.2*			
12mo	1314.6*	1325.6	1325.7	1446.2*	1466.3	1472.8
USD						
6mo	1268.9	1243.2	1220.5*			
12mo	2294.5*	2329.2	2322.2	2506.0*	2550.9	2549.5
GBP						
6mo	260.8*	266.2	262.6			
12mo	1405.1*	1441.1	1441.4	1588.2*	1630.0	1631.3
CHF						
6mo	1183.8*	1207.3	1185.7			
12mo	1407.4	1444.0	1406.4*	1390.1	1431.6	1388.9*

As well as in-sample data, the VECM methodology may be applied to test its predictive power in the determination of short term interest rates out-of-sample. In this case a VECM is fitted to a limited data sample and a prediction of the next days interest rates can be made using the determined parameters. This process may then be rolled over on a day by day basis.

As noted in the uni-variate case, the use of a threshold, in terms of the change in interest rates for the previous day can be effective as a filter. For the case of the multi-variate analysis, the value of the VECM relies on the fact that there is, in fact a consistent error in the response of one the yields to a new piece of information. As postulated above, if one considers the noise in the data to be consisted of two types. Firstly, there is random noise associated with bid-ask, liquidity concerns and flows. This noise is effectively random with zero mean. Secondly there is noise associated with the release of new information. There is noise associated with the surprise content of the news and noise associated with the uncertain response of the financial instrument to that noise. The use of a threshold for the change in the previous day's yields seeks to isolate the case where the change is dominated by the error correction mechanism and not by general market noise.

*Table 4: The m-BIC for three cross currency models, using 3mo LIBOR data from different currencies. The models are null hypothesis (Null), non-autoregressive VECM (NAR) and the full VECM (VECM) using EUR, USD, GBP and CHF data. The minimum m -BIC is identified using a “\*”*

	EUR 3mo			CHF 3mo			GBP 3mo		
	Null	NAR	VECM	Null	NAR	VECM	Null	NAR	VECM
EUR									
CHF	771*	822	783						
GBP	11.0	38.2	-2.1*	652*	690	663			
USD	510	508	450*	1006	981	951*	370	420	354*

In addition care has to be taken to ensure that the prediction has been made from a statistically significant fit. Thus the predictions are only used in cases

where the standard errors on the fitted parameters indicated significance at or greater than the 10% level. Whilst this does have the impact of reducing the number of data points, it does add an additional level of confidence on the significance of the predicted values.

From table 5 it may be seen that, consistent with the expectations hypothesis, the evolution of the longer maturity interest rate for the EUR, is to a limited extent influenced by the evolution of a shorter maturity interest rate in the same currency. Very clearly, however combining the results of the Diebold -Mariano and the Directional Accuracy tests, whilst the VECM has some power in determining the direction of the movement of the yield changes, it has very little power in determining both the direction and the magnitude of the change.

*Table 5: The Diebold-Mariano statistic and the Directional Accuracy Test for out-of-sample data produced using a non-autoregressive VECM (NAR) and a full VECM.*

Data1	Data2	Thold	Pts	SR	NAR DM	DA
EUR 3m	EUR 6m	1.5bp	115	57%	0.65	3.83
USD 3m	USD 6m	2.5bp	102	57%	-2.18	0.01
GBP 3m	GBP 6m	1bp	233	61%	-1.51	0.131
EUR 9m	EUR 1y	2bp	399	50%	-3.14	0.63
USD 9m	USD 1y	3bp	300	55%	-1.27	1.70
GBP 9m	GBP 1y	2bp	192	48%	-3.58	-0.63
EUR 1y	EUR 9m	2.5bp	180	52%	-3.45	1.08
USD 1y	USD 9m	4.0bp	156	60%	-1.76	2.32
GBP 1y	GBP 9m	2.5bp	225	48%	-5.28	-1.12
Data1	Data2	Thold	Pts	SR	VECM DM	DA
EUR 3m	EUR 6m	1.5bp	115	55%	-2.33	1.57
USD 3m	USD 6m	2.5bp	102	50%	-4.20	-0.77
GBP 3m	GBP 6m	1bp	233	61%	-0.85	3.16
EUR 9m	EUR 1y	2bp	399	50%	-3.95	0.338
USD 9m	USD 1y	3bp	300	55%	-4.73	1.22
GBP 9m	GBP 1y	2bp	192	48%	-3.68	-0.86
EUR 1y	EUR 9m	2.5bp	180	58%	-2.82	2.33
USD 1y	USD 9m	4.0bp	156	58%	-4.14	1.59

GBP 1y	GBP 9m	2.5bp	225	49%	-5.53	-0.228	
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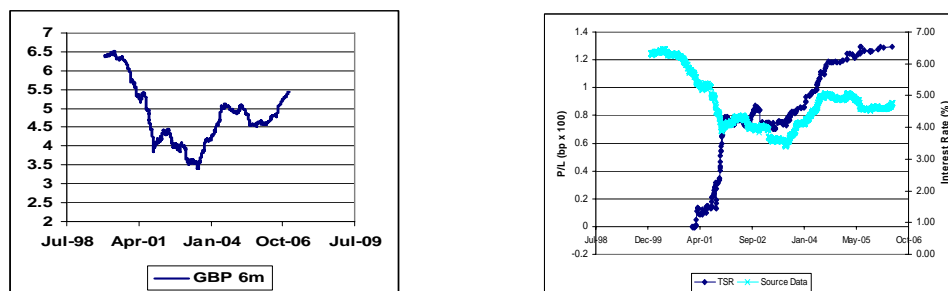
An issue arises when even longer maturity interest rates are considered. In this case it may be seen that the evolution of the shorter maturity interest rate, in this case the 9mo, is to a limited extent determined by the prior evolution of the 1yr rate. This is clearly not in agreement with the expectations hypothesis. A possible explanation relates to the liquidity of the two instruments, so that price discovery takes place in the more liquid instrument first, and that this is then used to determine the prices of less liquid, but related instruments. These two cases however indicate that the high frequency evolution of the yield curve can exhibit consistent non-linear behaviour that is not accounted for using current yield curve models.

The results presented in this section illustrate that the high frequency behaviour of short term interest rates exhibit behaviour that is significantly at odds with that predicted by the efficient market hypothesis. Both singular and combinatorial, the evolution of the rates is not only impacted by current information, but also but the legacy of past information, which has already had a discernable impact on the evolution of short term interest rates. This in turn will imply that the ability to determine the specific effects of surprise information on the yield curve is compromised to the extent that one cannot assume, a priori that only new information will impact the yield curve. This particular issue will be considered in a later paper where the impact of surprise economic information will be considered.

## 6: Trading Strategies

Having fitted the threshold regression and tested the fits versus insample data it is necessary to carry out out-of-sample fitting, to determine whether or not there is any real predictive power for this methodology. From table 2 it is observed that the results for six month GBP Libor have the most statistically significant DM and DA test results. These results were obtained using a TSR(1) model. As such it may be expected to provide the best performance in terms of out-of-sample predictions. The

results of applying such a strategy are shown in Figure 2. In this case the model makes statistically significant ( $SR = 63\%$ ,  $DA = 4.16$ ) predictions of the out-of sample next day movement of the GBP Libor rate. In addition the strategy shows the ability to make prediction independent of the trend direction of the market at the time of prediction.



*Figure 2: The time series history of GBP 6mo LIBOR and the returns of a strategy based on fitting a  $TSR(1)$  to the data and predicting the next day, out of sample market movement.*

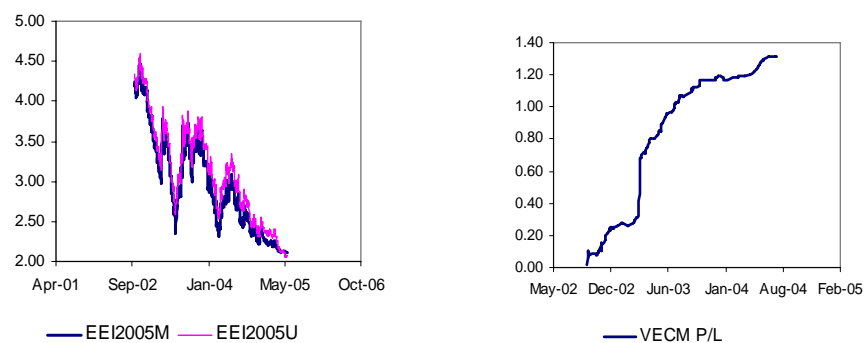
The simple  $TSR(1)$  model demonstrates a statistically significant ability to predict the direction of change of GBP six month interest rate using only the previous day's change as input data over a wide range of market conditions. Statistically, it does not depend on the ex-post trend of the market or on the local volatility.

In reality however it is not a trivial matter to actively trade LIBOR rates and in order to properly determine the success or otherwise of the strategy, it would be necessary to include funding and carry costs, which would need to be determined on an ex-ante basis. Whilst estimates of such costs have been included in the data presented above, there are still potential costs that may effectively nullify the predictive advantages gained from using the threshold autoregressive scheme. In order to further test the applicability of the models, they were fitted to 3 month EURIBOR futures closing price data. These have the advantages of being extremely liquid and extremely easy to trade, without the burden of significant additional costs. For all of the futures contracts used, no statistically significant results were found

when using the uni-variate models. The data covered the period of the September 1998 contract to the expiry of the March 2007 contract.

The multivariate approach was also applied to the futures data. The VECM models may be used to generate out-of-sample predictions for pairs of futures contracts. A standard VECM was fitted to the data and one day forward predictions were made for the relevant prices. These were compared with the realised next day closing prices. The whole system was then rolled forward one day and the process repeated. Periods of illiquidity at the start and expiry of the contract were excluded from the study to ensure the consistency of the results.

The results of a strategy based on the VECM model are given in Figure 3. It should be stressed that in these figures no allowances were made for bid-ask and for trading slippages and as such must be considered as idealised results.

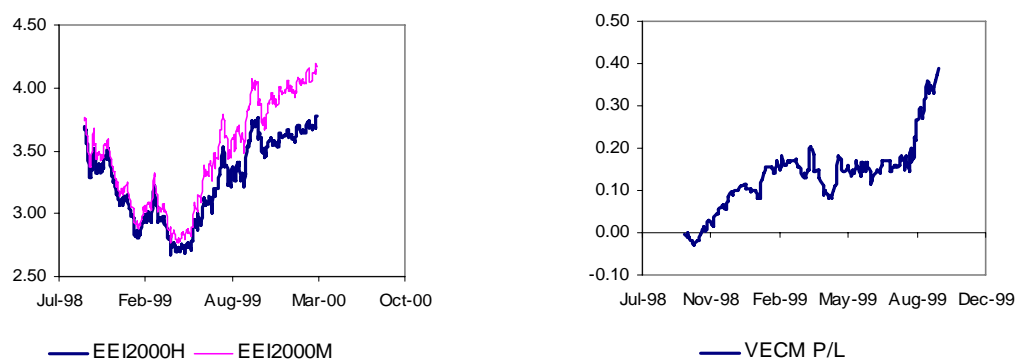


*Figure 3: The first graph shows the price history of the June 2005 and September 2005 EURIBOR futures contract from June 2002 to the expiry of the June 2005 contract. The second graph shows the results of an out-of-sample trading strategy based on the application of a VECM over the same time period.*

In addition it should be noted that the model specification needs to be adjusted for each currency pair under consideration and thus within the limited space available it is not possible to show data for all futures pairs. Across the data in question, statistically significant VECM fits were found in excess of 80% of the futures pairs considered.

In order to evaluate the efficiency of the trading strategy, it must be stressed, with reference to the data shown, that the VECM approach is not simply a trend

following proxy. Whilst a simple strategy of being long the futures contract would have produced a positive P/L outcome of approximately 180bps, this would have been achieved with a much higher volatility than that produced using the VECM approach. To further illustrate the point that the method does not depend on trending markets Figure 4 shows results obtained using the March 2000 and June 2000 futures contracts. In this case, over the period in question, expectations of future interest rates are first declined and then reversed. A simple trend following type model would have significant difficulty in producing positive results, given the high volatility of the contracts at this period in time.



*Figure 4: The first graph shows the price history of the March 2000 and June 2000 EURIBOR futures contract from September 1998 to the expiry of the March 2000 contract. The second graph shows the results of an out-of-sample strategy based on the application of a VECM over the same time period.*

## **Section 7: Discussion**

The data presented implies that for the major market economies the daily evolution of short term yields displays non-linear regression characteristics. There are a number of possible causes for this structure. The simplest explanation is that the spot rates are moving to their one day forward rates which given a persistent slope in the yield curve will lead to some form of autocorrelation. If this were the case, it would be an interesting result, because previous studies, conducted over longer time scales, indicate that the forward rates are not good predictors of spot rates. When



looked at in some detail the difference between the spot rate and its one day forward is extremely small compared to the volatility of the time series. As such the impact of the forward rate, whilst not possible to completely dismiss, will be small compared to the observed phenomenon. An example of this is given in figure 4, where the six month rate, its one day forward and the difference between the two are plotted for GBP data.

In addition no statistical relationship has been found between the differences predicted by the one day forward and the realised differences. As a final point, if the next day expectation of the rate was dominated by the forward rate, one would expect to see a significant greater number of accurate prediction events on a Monday, the two day weekend giving rise to a larger absolute difference than the other days. This is not observed in the data.

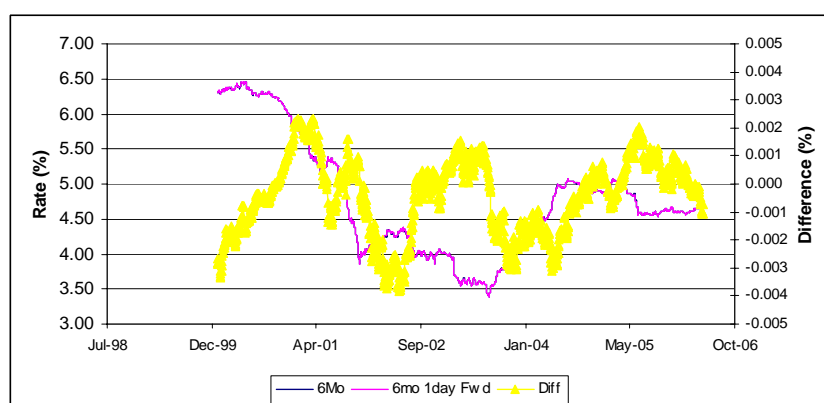


Figure 5: Six month spot and one day forward GBP interest rates

A second potential partial explanation is that the results merely reflect a significant constant drift in short term interest rates and all that is being observed is autocorrelation due to that drift term. In the financial markets this phenomenon is known as “the trend is your friend”. From finance theory this explanation is problematic as it seems incompatible with efficient markets theory. In addition, from the actual data it is clear there is some substantive non-linear process occurring that is not being accounted for in the drift hypothesis. The observed drift in the market is generally significantly smaller than the observed volatility. In the case of the EUR 3

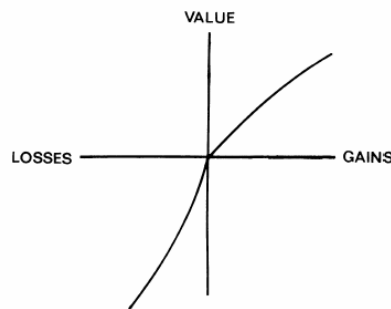
month Libor data the typical drift, when the data was actually observed to be drifting was  $\sim 0.2\text{bps/day}$  compared to a daily volatility of  $\sim 2.5\text{bps/day}$ . As such the impact of the drift would not expect to be significant on the daily outcome of the interest rate. In addition the models are observed to produce statistically significant accurate predictions at times when no drift is observed on an ex-post basis.

A third possible explanation, which can be closely related to the dominant drift hypothesis and which is similarly difficult to reject is that news events that cause significant market movements are themselves auto-correlated in their impact. This would imply that after a market moving piece of information is released, the market consistently underestimates its expectations of the next piece of information, which is of itself, market moving. In other words, the market when it receives new information moves to reflect that information. However it does not update its expectations of future information to come and thus when consistent information is received in the future, the market reacts to it as if were completely original, even though it is similar to the prior received information. Whilst this appears to be unlikely, it does not seem to be significantly more unlikely than a simple prescriptive strategy that seems to have some predictive power over the future movement of interest rates.

A fourth explanation is that the real impact of new information, that causes yields to move above a threshold, is consistently underestimated on the day of its announcement. This means that after the market has had time to digest the full implications of the news, it then adjusts further in the original direction that it moved in the first place. As the model appears to have predictive power in rising, falling and sideways moving markets, this under-reaction does not seem to have a bias based on the trend of the yield curve.

Whilst it is not possible to comprehensively reject any of the explanations that have been offered, it seems extremely unlikely that on their own, or together they are sufficient to explain the observed autocorrelation. In this light an alternative explanation is offered. A working hypothesis that partially explains the observed behaviour may be found from a simple application of prospect theory (Kahneman and

Tversky 1979). In prospect theory, the utility value of a gain is less steep than the negative utility value of a loss, Figure 3.



*Figure 6: The utility of an agent with respect to gains and losses according to prospect theory*

To apply this, consider the following situation; when the market receives new information, there will be an expectation of the appropriate response with a certain variance about that expectation, based on the ability to predict the actual response of the central bank (which sets base interest rates). If the market moves to that expectation, then its expected utility the following day will be negative, if prospect theory is applied. This is because there is a 50% chance of the market moving up or down from the expectation. However given that utility curves are steeper for losses than for gains, the net utility is negative. Thus in order to have zero net utility for the next days move, the market will move to a value that is less than its real expectation. The next day, having had more time to consider the new information, and if the previous days expectation is still appropriate, the market will move to the now confirmed original expectation. This will effectively cause autocorrelation in the markets movements. This approach will be developed in the following section.

## **Section 8: Prospect Theory and Market Utility**

In order to see the impact of the asymmetric utility of prospect theory on the price movements of market instruments, it is convenient to start with a simple staged

example. Consider the case where a trader is long the market with one unit of a commodity. After the release of some information, the trader has the expectation that in two days time the price level of the commodity will move from its original price “p” to a new price 2p. On the intervening day, the price of the commodity will be either 1.5p or 2.5p with 50% probability. In this case the utility of the trader may be given using prospect theory. For gains in price  $U = p_{t+1} - p_t$  and for losses in price  $U = \alpha * (p_{t+1} - p_t)$ , where  $\alpha$  represents the increased negative utility associated with losses.

In this case the expectation of the price of the commodity  $E(p_{t+2}) = 2p$  on both days, the issue that concerns us here is the path that the price of the commodity takes to get to the final price. Consider the utility of the trade for the two possible paths:

If the price moves to 1.5p on the first day and then onto to 2p the utility is as follows:

$$U_1 = (1.5p - p) + (2p - 1.5p) = p$$

On the other hand if the price moves to 2.5p on the first day and then back to 2p the utility is as follows

$$U_2 = (2.5p - p) + \alpha(2p - 2.5p) = p\left(1.5 - \frac{\alpha}{2}\right)$$

As can be seen from these two utilities  $U_1$  is greater than  $U_2$  for all values of  $\alpha$  greater than 1. So from the perspective of the trader, if he/she is interested in maximising their utility the first path will be taken under all circumstances. The trader prefers two wins in a row to a win and a loss. Effectively the same result as will be obtained if the trader is interested in maximising the Sharpe ratio. It may be demonstrated that under most simple conditions maximising utility under prospect theory will produce the same behaviour as would be expected from maximising Sharpe ratio. The cost functions are effectively transformations of each other. Thus this simple example illustrates that rational expectations do not necessarily lead to

symmetric paths in the presence of asymmetric utility for gains and losses, rather than only referencing the final state.

However the simple example is not easily applied to real market dynamics without significantly altering the assumptions behind the model. Obviously there are significantly more than two possible intermediate paths and it is further unlikely that the ex-ante expectation will in fact be realised. This brings us to the second case to be discussed. As the data that has been studied most intensely refers to short term interest rates, further discussion will be carried out with reference to these type of instruments.

It is a fundamental assumption that the values of all term interest rate instruments are determined, either directly or via an integrating process (at its most simple compounding), by future expectations of a base interest rate. At this point it is noted that credit effects are not being considered, however, except in pathological cases, this should not significantly alter the analysis. The fact that base interest rates are generally set by central banks gives a convenient starting point for considering the evolution of term interest rates and their expectations. Contrary to what has been the case throughout much of modern financial history, in the past decade the central banks have taken significant efforts into attempting to disclose to the market the parameters that they consider important in their setting of base interest rates. Firstly this may be seen in the actual mandates that they have been given by civilian governments. In the United States the Federal Reserve has a dual mandate of price stability (undefined) and economic growth (similarly undefined). In the United Kingdom the Monetary Policy Committee (MPC) has a defined inflation target defined as HICP of 2% +/- 1%. The MPC has shed further light on their deliberations through the minutes of the committee and the Quarterly Inflation Review. Other central banks display transparency to the markets to a greater or more normally lesser degree.

The principal point to be made is that interest rate markets, to some degree, have an appreciation of the appropriate response of a central bank to new economic information. It is this knowledge that impacts the responses of market participants to

new economic information (Gallmeyer, Hollifield et al. 2005). Consider the following situation. In an economy where the central bank is highly transparent, all market participants have a reasonable expectation of the central bank response to new information. However there is uncertainty around that expectation due to other related factors. An example of this is that under the influence of new information interest rate may be expected to increase by a certain amount. However if the yield curve steepens, this would decrease the expected increase, and if it flattened it would increase the expected move. However, all things being equal, both outcomes would lie around a single market expectation and that would be the rate to which the market would eventually move. In other words there is a knowable but not trivial to determine immediately, response function. For the purpose of this example the following notation will be used. At time  $t$  the relevant short term interest rate has a value  $r_t$ . Information is released at  $t$ , the result of which means that the market expectation of interest rates will be altered from  $r_t$  to value  $r_n$ . It will assumed that no further market moving information is released so that

$$E_{t+n}(r) = r_n \forall n > 0$$

As noted above, all market participants effectively have the same expectation, and we make the further assumption that there is some uncertainty about that value, effectively a volatility, which will be denoted by  $\sigma$ . It will further be assumed that all market participants have an asymmetric utility to gains and losses described above where the measure of greater loss aversion is represented by  $\alpha$ , where  $\alpha > 1$ . The sample probability distribution and utility graph are shown in figure 5. In this case we want to consider the likely market dynamics that may occur until the value of the post information rate is exactly realised. At time  $t+1$ , the market will have moved the reference interest rate to a value  $r_{t+1}$ . It is the purpose of this analysis to investigate how the value of  $r_{t+1}$  may be determined. The probability distribution of the rate will be represented by  $p(r_n, \sigma)$ . In this case the utility of the market makers will be given by

$$U_{t+1} = (r_{t+1} - r_t) + E_{t+1}(U_{t+2})$$

And the expectation at t+1 of the utility at t+2 will be determined by the expectation of the rate at t+2. Using the previous notation this will be given by:

$$E_{t+1}(U_{t+2}) = \alpha \int_{-\infty}^{r_{t+1}} (r - r_{t+1}) p(r_n, \sigma) dr + \int_{r_{t+1}}^{\infty} (r - r_{t+1}) p(r_n, \sigma) dr$$

This is effectively saying that if the market moves back from  $r_{t+1}$  at t+2 this will cause a loss, with enhanced negative utility, or it can move higher with positive utility. In efficient markets hypothesis, the assumption is that markets move to their expectation. In other words, today's rate is tomorrow's expectation. Prospect theory alters this assumption with the addition of asymmetric utility and the concept that markets move to a point where today's utility is tomorrow's utility. That is the expectation of tomorrow's utility is zero. Thus the market moves to a point where

$$E_{t+1}(U_{t+2}) = 0$$

$$\alpha \int_{-\infty}^{r_{t+1}} (r - r_{t+1}) p(r_n, \sigma) dr + \int_{r_{t+1}}^{\infty} (r - r_{t+1}) p(r_n, \sigma) dr = 0$$

In order to determine the appropriate level for the market at t+1 it is necessary to determine the level  $r_{t+1}$  where the above integral equation is zero. In order to investigate this condition, we make that assumption that the probability distribution of the rate is normal with mean  $r_n$  and volatility  $\sigma$ . In this case the condition becomes:

$$\frac{\alpha}{\sigma \sqrt{2\pi}} \int_{-\infty}^{r_{t+1}} (r - r_{t+1}) \exp\left(-\frac{(r^2 - r_n^2)}{2\sigma^2}\right) dr +$$

$$\frac{1}{\sigma \sqrt{2\pi}} \int_{r_{t+1}}^{\infty} (r - r_{t+1}) \exp\left(-\frac{(r^2 - r_n^2)}{2\sigma^2}\right) dr = 0$$

After some algebra this may be rewritten as

$$\alpha (r_n - r_{t+1}) + (\alpha + 1)$$

$$\left[ \frac{\sigma}{\sqrt{2\pi}} \left\{ \frac{\exp\left[1 - \frac{(r_{t+1} - r_n)^2}{2\sigma^2}\right]}{\left[1 - \frac{(r_{t+1} - r_n)^2}{2\sigma^2}\right]} \right\} + (r_n - r_{t+1}) \operatorname{erf}\left(\frac{r_{t+1} - r_n}{\sigma \sqrt{2\pi}}\right) \right] = 0$$

Where “erf” represent the error function. Whilst this is still not directly analytically solveable in terms of  $r_{t+1}$ , the equation may be linearised to give an idea of its fundamental characteristics:

$$(r_n - r_{t+1}) \propto (\alpha - 1) \cdot \sigma$$

Firsly it may be seen the  $r_{t+1}$  will always be less than  $r_n$  for all  $\alpha > 1$ .

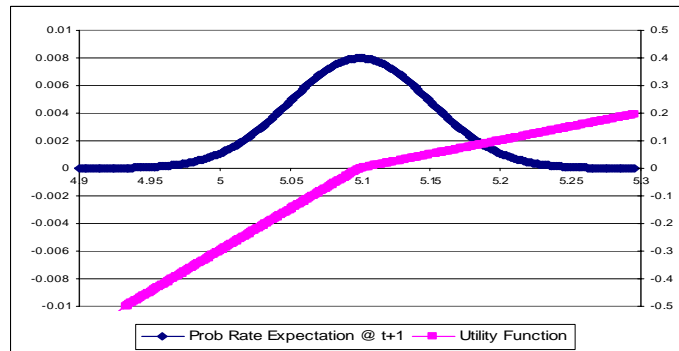




Figure 7: In this example the expectation of the rate is at 5.1%. Given the asymmetric utility function, the intermediate rate level giving zero expected utility for the subsequent days move is 5.086%.

When  $\alpha = 1$ ,  $r_{t+1} = r_n$ . Thus if the market has no real confidence in its expectation, then it will have no significantly larger loss aversion and so the market will move to its expectation, however unsure of it. Secondly as  $\alpha$  increases the distance by which  $r_{t+1}$  is less than  $r_n$  also increases. In other words, the greater the loss aversion, the more likely you are to set intermediate values significantly less than your expectation so as to ensure positive utility outcomes on subsequent days. In addition as the uncertainty around the expectation increases, the intermediate rate will move away from the expectation. These are the only parameters that impact the difference between the intermediate rate and the expectation. Thus in a market where there is a high level of transparency of the central bank response function, the intermediate difference is effectively a function of the loss aversion and the uncertainty about the response. Given a high level of transparency the uncertainty should be reasonably constant irrespective of the relevant information.

It is possible to solve the integral numerically and this has been done for a specific example and the results are presented in figure 6.

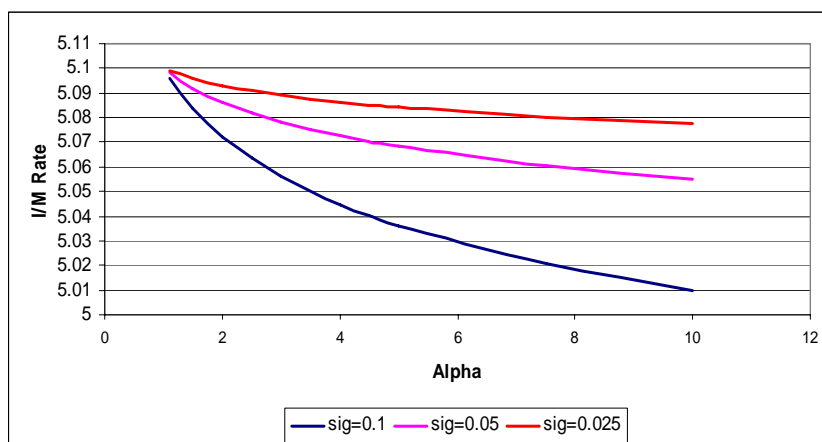


Figure 8: In this example the expectation is at 5.1%. The three sigmas are represented in basis point volatility.

As can be seen the difference between the intermediate rate and the expectation increases with both  $\alpha$  and  $\sigma$ . However there is not a strictly linear relationship and as  $\alpha$  increases the intermediate distance effectively reaches a stable value. Thus at the limit the difference between the intermediate rate and the expectation becomes a function of the uncertainty of the expectation, irrespective of the loss aversion.

The implication of this prediction is that, under conditions where the central bank has a high level of transparency of response to new information, the evolution of short term market determined interest rates should follow a sign regressive form. This is consistent with what has been observed in the GBP data.

However, it is clear that the assumptions necessary to derive the result above are not realistic for most central banks and thus it is necessary to consider a more general situation. Here we consider the case where on the release on new information, the different market participants will have different expectations, and each market participant has their own uncertainty about their expectation, and their own loss aversion, based on their certainty of their estimation. Those market participants who have no real confidence in their estimation will have  $\alpha = 1$  and thus may not need be considered in the further analysis. However given some transparency of the central bank the various expectations may be expected to be clustered around some central expectation and will have a probability distribution that will be represented by  $P(r_N, \sigma_N)$ , where  $\sigma_N$  is the measure of the uncertainty of the integrated expectation of all the markets participants. In this case the market will set an intermediate rate  $r_{t+1}$  and it is necessary to consider the factors that influence the actual value of this rate. This representation leads to complications for those market participants whose strongly held expectations ( $\alpha > 1$ ) are significantly different from  $r_N$ . This is because if the intermediate rate is greater than their expectation, they will effectively go short the market so that they may generate positive utility from their observation that the market has overshoot their expectation. Thus in this case it is necessary to consider the

utility of the different market participants from the initial release of the information.

In this case the utility function will have the form:

$$E_{t+1}(U_{t+2}) = \int_{-\infty}^{r_{t+1}} P(r_N, \sigma_N) \left[ \int_{-\infty}^{r_{t+1}} (r_{t+1} - r) p(r, \sigma) dr + \alpha \int_{r_{t+1}}^{\infty} (r_{t+1} - r) p(r, \sigma) dr \right] dr_N + \int_{r_{t+1}}^{\infty} P(r_N, \sigma_N) \left[ \alpha \int_{-\infty}^{r_{t+1}} (r - r_{t+1}) p(r, \sigma) dr + \alpha \int_{r_{t+1}}^{\infty} (r - r_{t+1}) p(r, \sigma) dr \right] dr_N$$

In this expression, without significant loss of generality, the different relevant market participants are assumed to have the same uncertainty around their expectations and the same loss aversion factor. The first integral represents those who believe that the intermediate rate  $-r_{t+1}$  is an overshoot of the correct level and the second integral represents those who believe that the intermediate level is an underestimate.

In attempting to understand the value of this equation it should be noted that the integrated expected utility of involved market participants is strictly positive. In other words irrespective of the value that  $r_{t+1}$  takes, the different participants will be able to construct a trading strategy by which they may expect to have positive utility. Thus in this model it will not be possible to have an  $r_{t+1}$  that gives the intellectually attractive expected utility of zero. However it is clear that the further away that  $r_{t+1}$  is from  $r_N$  on the downside, the lower the net expected utility. This is because the lower the intermediate rate is, the fewer participants who will expect to get additional utility from (from their perspective) the reversal in the market (They profit on the way up and on the way down).

Whilst not attempting to directly determine  $r_{t+1}$  from this integral it is possible to draw some conclusions if it is assumed that the uncertainty of the net market

expectation –  $\sigma_N$  is a function of the difference between the original rate,  $r_t$  and the market expectation,  $r_N$ . The justification behind this assumption is that given that the market is not extremely confident of its expectations, then a piece of new information that is significantly market moving, will have a correspondingly significant uncertainty about that expectation. If this is the case then it is possible to generate an estimation of the level of the intermediate rate,  $r_{t+1}$ . As net market expected utility is reduced the further away the intermediate rate is from the market expectation, then it will be set at a level a number of standard deviations away from the expectation.

$$r_N - k_1 \sigma_N = r_{t+1}$$

where  $k$  is the appropriate multiplier. If you make the assumption that  $\sigma_N$  is a linear function of the difference between  $r_t$  and  $r_N$ :

$$\sigma_N = k_2 (r_N - r_t)$$

where  $k_2$  is the appropriate multiplier. Combining the two relationships, it may be shown that

$$(r_N - r_{t+1}) = k_3 (r_{t+1} - r_t)$$

where  $k_3$  is a constant. In other words, under the assumptions stated above, this approach implies that short term interest rates rates will exhibit autocorrelation. The level of autocorrelation is a function of the degree of transparency of the relevant central bank, and the level of uncertainty in the “knowability” of the response of the central bank to new information.

A consequence of this hypothesis is that the more transparent a central bank, the higher the level of autocorrelation. This is reflected in the observation that the diagnostic tests give the highest level of significance to the GBP data and the lowest to CHF. In addition, it would be expected that this effect should diminish with

maturity of the interest rate. This is because the shorter the maturity of the interest rate, it would be expected that market participant would have a higher level of confidence in their ability to accurately assess the response of a central bank to a new piece of information. As such, it would be expected that short term interest rates will display a higher degree of auto-regressive behaviour, than longer term interest rates. This is observed in the data. However consistency does not imply evidence and further work needs to be carried out.

## **9: Conclusions**

This paper has described work that has been carried out on daily sampled interest rate data in an attempt to describe the processes that impact the evolution of interest rates over the short term independent of exogenous news such as economic information.

Consistent non-linear autoregressive behaviour has been seen across a range of currencies in the front end of the yield curve. In addition, part of the evolution of different maturities of the yield curve may be effectively modelled using a vector error correction mechanism. It should be stressed that this behaviour is not consistent with the efficient markets hypothesis for the evolution of interest rates. A caveat to this statement is that expectation theory would indicate that the evolution of long maturity interest rates should depend on that of shorter rates. The theory however does not indicate that there should be a differential time lag in the communication of the changing expectations. In addition, given that the results suggest that different intra-maturity responses are seen across different currencies indicate that currency specific effects are at work. Given that the data under study is short term interest rates, it suggests that the different behaviour may be influenced by the utility function of the relevant central bank and the markets response to its perception of that utility and how it is implemented. This is a very relevant result in a financial environment where central banks are known to study the influences of their decisions on the marketplace, and then potentially alter their responses to economic data based on that information. A more complete consideration of these effects will be the considered in

later work. The autoregressive behaviour, either uni-variate or multivariate may be used to construct a statistically significant positive expectation trading strategy.

In addition, a heuristic argument, based on prospect theory has been advanced that explains the observed autocorrelation in terms of asymmetric utility associated with gains and losses and the confidence level as applied to central bank actions. The predictions of this simple model are found to be consistent with that observed in the data.

## References

- Ang, A. and M. Piazzesi (2003). "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables." Journal of Monetary Economics **50**(4): 745-787.
- Ang, A., M. Piazzesi, et al. (2006). "What does the yield curve tell us about GDP growth?" Journal of Econometrics **131**(1-2): 359-403.
- Balduzzi, P., E. J. Elton, et al. (2001). "Economic News and Bond Prices: Evidence from the U.S. Treasury Market." The Journal of Financial and Quantitative Analysis **36**(4): 523-543.
- Campbell, J. Y. and J. Ammer (1993). "What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns." The Journal of Finance **48**(1): 3-37.
- Campbell, J. Y. and R. J. Shiller (1991). "Yield Spreads and Interest Rate Movements: A Bird's Eye View." The Review of Economic Studies **58**(3): 495-514.
- Chan, K. S. (1993). "Consistency and Limiting Distribution of the Least Squares Estimator of a Threshold Autoregressive Model." The Annals of Statistics **21**(1): 520-533.
- Clarida, R., J. Gali, et al. (1999). "The Science of Monetary Policy: A New Keynesian Perspective." Journal of Economic Literature **37**(4): 1661-1707.
- Cox, J. C., J. E. Ingersoll, Jr., et al. (1985). "A Theory of the Term Structure of Interest Rates." Econometrica **53**(2): 385-407.
- Dai, Q. and K. Singleton (2003). "Term Structure Dynamics in Theory and Reality." The Review of Financial Studies **16**(3): 631-678.
- Dickey, D. A. and W. A. Fuller (1979). "Distribution of the Estimators for Autoregressive Time Series With a Unit Root." Journal of the American Statistical Association **74**(366): 427-431.
- Diebold, F. X. and R. S. Mariano (1995). "Comparing Predictive Accuracy." Journal of Business & Economic Statistics **13**(3): 253-263.
- Driessen, J., B. Melenberg, et al. (2003). "Common factors in international bond returns." Journal of International Money and Finance **22**(5): 629-656.

- Ederington, L. H. and J. H. Lee (1993). "How Markets Process Information: News Releases and Volatility." The Journal of Finance **48**(4): 1161-1191.
- Estrella, A. (2005). "Why Does the Yield Curve Predict Output and Inflation." Federal Reserve Bank of New York.
- Estrella, A. and G. A. Hardouvelis (1991). "The Term Structure as a Predictor of Real Economic Activity." The Journal of Finance **46**(2): 555-576.
- Fama, E. F. (1984). "Forward and spot exchange rates." Journal of Monetary Economics **14**(3): 319-338.
- Gallmeyer, M. F., B. Hollifield, et al. (2005). "Taylor rules, McCallum rules and the term structure of interest rates." Journal of Monetary Economics **52**(5): 921-950.
- Gospodinov, N. (2005). "Testing For Threshold Nonlinearity in Short-Term Interest Rates." Journal of Financial Econometrics **3**(3): 344.
- Granger, C. W. J. and T. Terasvirta (1999). "A simple nonlinear time series model with misleading linear properties." Economics Letters **62**(2): 161-165.
- Green, T. C. (2004). "Economic News and the Impact of Trading on Bond Prices." The Journal of Finance **59**(3): 33.
- Gürkaynak, R. S. S., Brian; Swanson, Eric. (2005). "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models. ." The American Economic Review **95**(1): 12.
- James J, N. W. (2000). Interest Rate Modelling. Chichester, Wiley.
- Kahneman, D. and A. Tversky (1979). "Prospect Theory: An Analysis of Decision under Risk." Econometrica **47**(2): 263-292.
- Kyle, A. S. (1985). "Continuous Auctions and Insider Trading." Econometrica **53**(6): 1315-1335.
- Lutkepohl, H. (2005). New Introduction to Multiple Time Series Analysis. Berlin, Springer-Verlag.
- Pesaran, M. H. and A. Timmermann (1992). "A Simple Nonparametric Test of Predictive Performance." Journal of Business & Economic Statistics **10**(4): 461-465.
- Piazzesi, M. (2001). "An Econometric Model of the Yield Curve with Macroeconomic Jump Effects " NBER Working Paper Series(Working Paper 8246).



Schwarz, G. (1978). "Estimating the Dimension of a Model." The Annals of Statistics **6**(2): 461-464.

Tong, H. (1990). Non-Linear Time Series - A Dynamical System Approach, Oxford University Press.