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Geary WP2022/10
Oct 17, 2022

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Abstract

In this paper the impact of investment horizon on asset co-skewness is examined both empirically and theoretically. We first detail a strong horizon-based estimation bias for co-skewness. An asset that has positive co-skewness at one horizon may have negative co-skewness for others. This phenomenon is particularly evident for small-capitalization stocks. We then propose a theoretical model to estimate long-horizon co-skewness using data observed at the shortest horizon, which emphasizes the role of adjustment delays in the pricing of market-wide information among securities. Co-skewness is only found to be priced in the cross-section of stock returns for a small range of short-horizons, calling into question the universal validity of the three-moment model.

Keywords: Co-skewness; The Horizon Effect; Intertemporal Correlation; Asset Pricing

JEL Classification: G10, G12, G14

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This publication has emanated from research conducted with the financial support of Science Foundation Ire- land under Grant Number 16/SPP/33 and 17/SP/5447 and 13/RC/2106 P2. Chenglu Jin also gratefully acknowledges grants from the key program of the National Natural Science Foundation of China (NSFC No. 72101229 and No. 71631005). We gratefully acknowledge comments by Akhtar Siddique, Alexandros Kostakis, Utpal Bhattacharya, Wolfgang Bessler, Carol Alexander, along with participants at the 2016 INFINITI conference on international finance, 4th Young Finance Scholars Conference, University of Sussex and IFABS Asia 2017 Ningbo Conference.

1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) has been supplemented in various ways to better explain expected asset returns. In particular, Kraus and Litzenberger (1976, 1983) and Harvey and Siddique (2000) have provided evidence that systematic skewness, often referred to as co-skewness or gamma, further characterizes the risk of an individual security relative to the market. The importance of accounting for co-skewness both in asset pricing (Smith, 2007; Kostakis et al., 2012; Lambert and Hubner, 2013; Kalev et al., 2019) and optimal portfolio allocation (Martellini and Ziemann, 2010; Jondeau and Rockinger, 2012) has been documented. The existing literature has, however, evaluated co- skewness over a range of arbitrarily chosen horizons, since the single-period model underpinning most studies considering co-skewness is silent on the appropriate length of an investment period. Monthly or daily returns are widely used without particular justification.[1] In this paper, we provide an empirical and theoretical assessment of the estimation and pricing role of co-skewness across multiple horizons.

A long line of research has considered the horizon effect on the estimation of financial parameters.[2] Initially, in a framework where investors are assumed to be heterogeneous with respect to their investment horizon, a number of papers documented that the systematic risk (beta or β) of an asset or a portfolio changes as the horizon is extended under the single-period Sharpe-Lintner CAPM (Cohen et al., 1980; Hawawini, 1980b; Handa et al., 1989; Gencay et al., 2005; Perron et al., 2013; Bandi et al., 2021). Recently, a considerable body of work has also yielded similar horizon effects, for example, in risk–return relationships (Jacquier and Okou, 2014), correlation estimation (Conlon et al., 2018), common risk factors (Kamara et al., 2016; Brennan and Zhang, 2019), measuring financial connectedness ((Barunik and Kehlik, 2018)), and in demonstrating a term structure associated with downside risk measures (Engle, 2011; Guidolin and Timmermann, 2006). Only limited consideration has been given, however, to the effect of the investment horizon on the estimation and pricing of co-skewness.

Motivating our study, Fama and French (2018) provide evidence that the distribution of short-horizon returns is skewed and leptokurtic relative to the normal distribution but that these characteristics are altered for longer horizons. Holding mean and variance constant, prudent investors should prefer assets for which returns are right-skewed, relative to those that are left-skewed (Harvey and Siddique, 2000). Accordingly, assets with negative co-skewness to the market portfolio, with a resultant decrease in a portfolio's skewness, require higher expected returns; and vice

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[1] We have examined 47 papers investigating asset co-skewness, published in top peer-reviewed journals including the Journal of Finance, Journal of Financial Economics, Review of Financial Studies, and Management Science since 2000. 30 of these studies use monthly and 8 daily returns only, while 9 consider returns measured using two or more horizons. For example, Conrad et al. (2013) test the impact of ex-ante skewness on expected stock returns using both daily and monthly returns. Langlois (2020) empirically investigate the respective roles of systematic and idiosyncratic skewness in explaining expected stock returns using daily and monthly returns.

[2] The 'horizon effect' has, inter alia, been referred to in the literature as: the intervaling effect, the frequency problem, the investment horizon problem, and the holding period problem, amongst other.

versa. In this paper we present persuasive empirical evidence that co-skewness is highly sensitive to the length of the investment horizon and highlight the possibility that portfolios with positive co-skewness in one horizon may have negative co-skewness when measured using another horizon. More precisely, the signs of estimated co-skewness parameters are prone to reversal across differing horizons for size-sorted portfolios, an empirical finding which has not been reported in the literature previously.[3]

An important contribution of our study is the provision of theoretical and economic underpinnings for the horizon effect on estimated co-skewness. The literature documents two primary explanations for the horizon effect. First, studies focus upon the estimation bias which occurs when using different lengths of the investment horizon in estimating financial parameters. For example, research suggests that there may be delays in price adjustment for certain stocks to market-wide news (Lo and MacKinlay, 1990; Brennan et al., 1993; Hou and Moskowitz, 2005; Zhang, 2006). The heterogeneous speed among firms in releasing information and the adjustment of stock prices to market-wide information induce cross-serial correlation in security returns, which may also lead to autocorrelation in market index returns. Moreover, Levhari and Levy (1977) find that, even if returns are independent and identically distributed over time, the link between n-period returns and 1-period returns results in “a complex relation” between 1-period betas, $\beta i(1)$, and n-period betas, $\beta i(n)$. Brennan and Zhang (2019) confirm the significant role of the multiplicative relation. Estimation using short horizon data may also be biased due to thin trading as identified by Dimson (1979) and Scholes and Williams (1977), which leads to a lack of synchronization between observed security prices and the market portfolio.

Accordingly, following this literature we develop a model of the horizon effect on co- skewness, theoretically indicating that co-skewness estimated at any given T-period horizon can be expressed as a function of the daily co-skewness, the length T of the return horizon in days, and the security’s intertemporal cross-correlation to the market as well as market autocorrelations in unit returns.[4] We show that our model produces accurate long-horizon estimates of co-skewness using only data from the shortest horizon. Furthermore, our model affirms that the estimation bias resulting from intertemporal correlations among security returns is an important source of the horizon effect on co-skewness. We provide evidence that the horizon effect on co-skewness is more conspicuous when higher order serial correlations are accounted for. Moreover, for a given order of serial correlation, increasing or decreasing the

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[3] While the impact of horizon on co-skewness has not previously been examined, skewness of asset returns has been shown to be horizon dependent (Hawawini, 1980b; Lau and Wingender, 1989) and to suffer from sampling error (Lau et al., 1989).

[4] In the context of this paper, intertemporal correlations refer to the range of non-contemporaneous lagging and leading correlations between assets and the market, including those involving quadratic terms.

magnitudes of the intertemporal correlation coefficients is also shown to induce estimation bias in co-skewness. These results help to explain our empirical findings for size-sorted portfolios.[5]

A second possible explanation for the horizon effect is linked with a line of literature which documents evidence on the variations in investment horizons across investors and over time, from a behavioural perspective. For example, Aıt-Sahalia and Brandt (2001) find that the selection of optimal portfolios and the corresponding weights vary across investment horizons. Dierkes et al. (2010) investigate interactions between investment preferences and horizon, and find that investing in stocks results in higher utility for investors with long investment horizons. Furthermore, clientele with different trading frequencies may have different perceptions of risk (Andries et al., 2019; Hur and Singh, 2017) and heterogeneous preference for skewness (Mitton and Vorkink, 2007). If investors are risk-averse, prudent and temperate, literature suggests that firms whose returns exhibit negative co-skewness should require higher premia relative to those with positive co-skewness (Harvey and Siddique, 2000; Kostakis et al., 2012). Thus, investors who trade frequently may have higher risk-tolerance and prefer to invest in securities with larger negative co-skewness to seek a higher risk premium, while longer-horizon investors may be willing to accept a low premium and only pursue securities with low co-skewness.

Relating to this idea of heterogeneous investment horizons, one might expect the horizon to match that of an institutional investor. As argued by Benartzi and Thaler (1995), a one-year horizon is the most plausible choice for the investment evaluation horizon. In this sense, our findings may provide further evidence for the arguments of Dittmar (2002), Post et al. (2008) and Post and Levy (2005) against the interpretation of the co-skewness premium as providing support for the three-moment model. The estimation methods adopted by early empirical studies of the three-moment model, including that of Harvey and Siddique (2000), do not restrict the pricing kernel to be globally decreasing, or the representative investor to be globally risk averse. Post et al. (2008) indicate that imposing global risk aversion removes the explanatory power of co-skewness. Looking at horizons longer than the traditional one-month interval considered in the literature, we provide additional evidence that co-skewness is not priced in the cross-section, adding to the questions over the empirical validity of the three-moment model. In the context of Kamara et al. (2016), the contrasting importance of co-skewness at different horizons might be attributed to a clientele effect, whereby short-run investors are compensated for exposures to shocks to which

[5] Theobald and Yallup (2004) suggest that the speed of pricing adjustment for larger firms is greater than that of smaller counterparts. The authors establish a lead/lag relationship and demonstrate that large firms have faster speeds of adjustment than small firms. Their results are consistent with various other papers, such as Jegadeesh and Titman (1995) and Lo and MacKinlay (1990), which have established and investigated the lead/lag effects across size sorted portfolios and show that returns of large capitalization stocks lead those of small capitalization stocks. This supports our finding that larger firms have a relatively small horizon effect on co-skewness.

long-run investors are less sensitive. Furthermore, Neuhierl and Varneskov (2021) provide evidence that low and high-frequency state vector risk is differentially priced.

Our theoretical model and related empirical findings also highlight that, while intertemporal cross-correlation and auto-correlation help to drive the horizon dependence of co-skewness, the horizon effect is present even without such characteristics. Previous theoretical work linked the horizon effect in beta to frictions, manifesting as intertemporal cross-correlation and auto- correlation. Hawawini (1980b) proposes a model to explain the relation between estimated betas and intertemporal cross-correlations, suggesting that there is no horizon effect on beta under the independence assumption in an efficient market. Our model, however, shows that the length of the investment horizon plays a significant role, indicating a "scaling law" of co-skewness in the absence of intertemporal correlation. This reinforces and provides a theoretical explanation for the heterogeneity in the empirical validity of the three-moment model described.

Finally, given the evidence of the horizon effect on co-skewness estimation, we examine the implications of this effect for higher-order asset pricing. We assess whether the cross-sectional variation in asset returns can be explained by exposure to co-skewness using the Fama and MacBeth (1973) two-step method. Specifically, we examine the pricing role of co-skewness across a variety of horizons. Co-skewness is found only to be significantly priced for horizons ranging from 2-days to one-month and not at longer horizons. In the context of our theoretical model indicating a scaling law of co-skewness, one needs to be careful in interpreting this as motivation for an optimal horizon for co-skewness. Instead, as documented by Fama and French (2018), asset returns tend to be more normally distributed when longer-horizon returns are considered, perhaps diminishing the potential for a co-skewness risk premium at such horizons.

Our findings for the impact of the sampling horizon on co-skewness are important for both portfolio selection and asset pricing. As proposed earlier, investors' preference for positive skewness typically leads to a desire for assets with positive co-skewness, representing those with higher probabilities of extreme positive outcomes for a security relative to market re- turns. Our results suggest that an asset which is selected for a portfolio based on its positive co-skewness using one investment horizon may have negative co-skewness at another horizon, with resultant implications for asset and portfolio selection (higher-order moments have been considered for portfolio optimization by Post and Kopa (2017), Martellini and Ziemann (2010), Guidolin and Timmermann (2008) and Patton (2004), for example.) Our empirical and theoretical findings convey a word of caution for empirical researchers who estimate asset co-skewness, especially for stocks with extreme firm size (largest or smallest).

The paper is structured as follows. Section 2 introduces the asset pricing implications of co-skewness from first principles and develops our model of horizon-dependent co-skewness. Section 3 describes the data employed and presents empirical results relating to the estimation of co-skewness coefficients. Section 4 presents further tests on the pricing of co-skewness factor and its relation to the horizon effect. Concluding remarks are given in Section 5.

2. Co-skewness and Modelling Process

The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), while useful in explaining the relationship between financial risk and return, has not been found to adequately explain the cross-section of stock returns. As noted by Brennan and Zhang (2019), the CAPM is a single-period model, where the horizon or time period is not specified.

Kraus and Litzenberger (1976) and Harvey and Siddique (2000) extend the static CAPM to nonlinear forms of the risk-return trade-off by considering systematic skewness, developing the notion that moments of returns other than variance are relevant to maximizing investors' expected utility. Under this framework a risk averse and prudent investor will have preference for a positively skewed portfolio. An asset with negative systematic skewness, or co-skewness, may not be selected, as this may result in a more negatively skewed portfolio. The implication for asset pricing is that the required risk premium on a stock is higher if co-skewness is negative. Harvey and Siddique (2000) provide empirical evidence of an economically important co-skewness risk premium and show that conditional skewness helps explain the cross-section of stock returns.

Early studies on the co-skewness premium, including Harvey and Siddique (2000), do not restrict the pricing kernel to be globally decreasing or require the representative investor to be globally risk averse. Imposing these restrictions, Post et al. (2008) show that the explanatory power of co-skewness is reduced considerably. Poti and Wang (2010) reconcile the empirical evidence, showing that the implied risk aversion coefficient must be implausible high for co-skewness to help explain stock returns. While Post et al. (2008) examine the impact of the return interval on the bounds of the gamma premium, no paper has examined the impact of horizon in detail or developed a model to explain any horizon disparity in co-skewness estimates.

In this paper, we build upon the extensive literature examining the effect of investment horizon on beta (for example, Perron et al. (2013), Handa et al. (1989), Cohen et al. (1983a), Hawawini (1980b) and Scholes and Williams (1977)) and determine whether co-skewness presents a similar phenomena. If co-skewness is horizon dependent, both the non-linear pricing kernel or portfolio selection with skewness may also be affected by the choice of the investment horizon.

2.1. Estimation of Co-skewness

We follow the approach proposed by Harvey and Siddique (2000) for the estimation of co- skewness. To estimate the degree of horizon-dependent co-skewness, we repeat the estimation procedure using returns observed at different horizons, and thus have several samples of co- skewness estimates. More precisely, we first employ the CAPM regression using a rolling window of 15-year excess returns for share *i*:

$$RiT,t - RfT,t = \alpha iT,t + \beta iT,t(RmT,t - RfT,t) + \varepsilon iT,t, \quad (1)$$

to extract the residuals $\varepsilon iT,t,$ which are, by definition, orthogonal to the excess market returns. RiT,t and R_mT,t represent T - day asset returns and market returns, respectively. Therefore, these residuals are net of the covariance (beta) risk, but still incorporate co-skewness risk.

Using the residuals from Equation 1, Harvey and Siddique (2000) estimate standardized co-skewness for share i at horizon T, namely $\gamma iT,t,$ using a 5-year window up to time t as:

$$\gamma_{\mathrm{iT,t}} = \frac{E[\epsilon_{\mathrm{iT,t}}\epsilon^2_{\mathrm{mT,t}}]}{\sqrt{E[\epsilon^2_{\mathrm{iT,t}}]E[\epsilon^2_{\mathrm{mT,t}}]}}, \quad (2)$$

where $\varepsilon iT,t = [RiT,t - RfT,t] - [\alpha iT,t + \beta iT,t(RmT,t - RfT,t)]$ is the residual from Equation 1 and $\varepsilon mT,t$ is the deviation in the excess market return for month t from the average value over the corresponding window.

Notably, other alternatives to the Harvey and Siddique (2000) co-skewness estimator exist. In their seminal paper, Kraus and Litzenberger (1976) introduced an unconditional estimator which does not have the orthogonality property associated with the Harvey and Siddique (2000) estimator. The later point is important in the context of the present work, as the Harvey and Siddique (2000) approach allows us to disentangle the horizon effect of co-skewness from that of beta, while providing a conditional version of the original Kraus and Litzenberger (1976) three-moment model. Moreover, as detailed by Harvey and Siddique (2000), their estimator is unit free and analogous to a factor loading. While the focus here is on the commonly-examined Harvey and Siddique (2000) estimator, we assess the robustness of our findings to the use of the Kraus and Litzenberger (1976) approach in Appendix D. Other approaches to estimate co-skewness, while not investigated here, have also been proposed including the predictive ordering method of Langlois (2020) and shrinkage approach of Boudt et al. (2020).

Although the literature has not considered in detail the statistical small-sample properties of the empirical estimators of co-skewness, analogous statistical properties have been examined for skewness estimators. Eberl and Klar (2020) examine the asymptotic properties of various empirical counterparts of theoretically motivated skewness estimators. The standardized central third moment, relating to the co-skewness estimator discussed here, is shown to have mediocre behaviour, especially for small samples. Contrasting the conventional coefficients of skewness and kurtosis with robust counterparts, Kim and White (2004) demonstrate the susceptibility of the conventional estimators to outliers in the data. These findings act as a further motivation to assess the horizon impact on co-skewness, based upon the weakened influence of outliers at long horizons. Our findings act as further evidence for the empirical challenges involved in the estimation of co-skewness using commonly used approaches.

2.2. Decomposition of Co-skewness: the Sum of Intertemporal Cross-covariances

A substantial literature has documented the horizon effect on both security alphas (Boguth et al., 2016) and betas (see, for example, Handa et al. (1989) and Hawawini (1983)), but the impact on co-skewness remains an open question. Given that residuals, $\varepsilon iT, t$, can be written as a function of both α and β, as shown in Equation 1, this, in isolation, should lead to co- skewness being horizon-dependent. In this paper, however, we wish to demonstrate the horizon dependence of co-skewness, itself related to intertemporal characteristics associated with return time series, independent of the horizon effect on alphas and betas. Similar to Cohen et al. (1983b), we develop a model to relate the horizon effect of co-skewness to characteristics of autocovariance and intertemporal cross-covariance, in light of the frequent documentation of these characteristics for financial time series in the literature (Cohen and Frazzini, 2008; Lo and MacKinlay, 1990).[6] We acknowledge that there may be multiple fundamental drivers of such horizon effects such as clientele, investment horizon preferences and estimation bias. Many of these suggested sources may manifest as autocorrelation and intertemporal correlation in the underlying data and, which, in turn, impact the estimation of co-skewness. Our approach has the benefit of allowing us to garner further insight into the estimation of co-skewness at various horizons in a frictionless environment. To this end the following proposition details the relationship between long-run co-skewness and characteristics estimated at short-run horizons.

Proposition 1. *Given an observed empirical distribution of logarithmic stock returns* $R_{iT,}$ *and market returns* $R_{mT,}$ *at horizon T the estimated co-skewness for stock i can*

[6] While our model can be expressed in terms of intertemporal cross-correlation terms, an exposition in terms of intertemporal cross-covariance provides equivalent insights, we follow this route for brevity.

be written as a function of unit returns $R_{i1,}$ and $R_{m1,}$ the length of the horizon the intertemporal auto-covariance of stock returns and market returns the intertemporal cross-covariance between returns of the stock and squared market returns and the estimated betas at horizon T and is given by:

$$\gamma_{\mathrm{iT,t}} = \frac{cov(\epsilon_{\mathrm{iT,t}},\epsilon^2_{\mathrm{mT,t}})}{\sigma(\epsilon_{\mathrm{iT,t}})\cdot var(\epsilon_{\mathrm{mT,t}})}$$
$$= \frac{\sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{\mathrm{i1,t\text{-}j}},R_{\mathrm{m1,(t\text{-}k)}}\cdot R_{\mathrm{m1,(t\text{-}l)}}) - \hat{\beta}_{\mathrm{iT,t}}\sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{\mathrm{m1,t\text{-}j}},R_{\mathrm{m1,(t\text{-}k)}}\cdot R_{\mathrm{m1,(t\text{-}l)}})}{\sqrt{\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}[cov(R_{\mathrm{i1,(t\text{-}k)}},R_{\mathrm{i1,(t\text{-}u)}}) - \hat{\beta}_{\mathrm{iT,t}} cov(R_{\mathrm{i1,(t\text{-}k)}},R_{\mathrm{m1,(t\text{-}u)}})]}\cdot\sum_{k=0}^{T-1}\sum_{u=0}^{T-1} cov(R_{\mathrm{m1,(t\text{-}k)}},R_{\mathrm{m1,(t\text{-}u)}})} \quad (3)$$

Proof: See Appendix A.

Equation 3 demonstrates that co-skewness at horizon T can be expressed as a function of short-run returns, $R_{i1,}$ and $R_{m1,}$, accounting for characteristics of auto-covariance and intertemporal cross-covariance. In Appendix A we show that α has no impact on the estimation of co-skewness and $\beta_{iT,t,}$ can be expressed as a function of short-run characteristics using the following expansion, as suggested in the literature:

$$\hat{\beta}_{\mathrm{iT,t}} = \frac{cov(R_{\mathrm{iT,t}},R_{\mathrm{mT,t}})}{var(R_{\mathrm{mT,t}})} = \frac{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{\mathrm{i1,(t\text{-}k)}},R_{\mathrm{m1,(t\text{-}u)}})}{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} cov(R_{\mathrm{m1,(t\text{-}k)}},R_{\mathrm{m1,(t\text{-}u)}})}. \quad (4)$$

2.3. The Long-Run Horizon Effect

Cohen et al. (1983a) documents that the estimated beta would converge to a "true" beta when the horizon is sufficiently lengthened, since returns measured using longer-horizons are less affected by serial correlations. Hawawini (1980a) suggests that if there is neither autocorrelation in market returns nor intertemporal cross-correlation between securities' returns and the market returns, beta is invariant to the length of the investment horizon. In a similar sense, we derive the following proposition to understand the long-horizon implications for co-skewness in a frictionless environment:

Proposition 2. *Given a stationary distribution of observed returns and assuming the absence of intertemporal auto-covariances and cross-covariances among asset returns, the magnitude of intertemporal estimates of unconditional co-skewness are inversely related to the square-root of the horizon T and co-skewness estimated using unit returns as $\gamma_{iT,t}$ follows:*

$$\gamma_{iT,t} = \frac{\gamma_{i1,t}}{\sqrt{T}}. \quad (5)$$

Proof: See Appendix B.

Equation 5 is analogous to the square root of time scaling law associated with risk and the related rule proposed for skewness (Lau and Wingender, 1989). The former has also been shown to be perturbed by serial dependence in the underlying time series (Wang et al., 2011).

In Equation 5, we demonstrate that the horizon effect on co-skewness may exist even in an efficient market with no price delay. Furthermore, Equation 5 shows the horizon effect on co-skewness is not merely a consequence of the horizon effect associated with beta and alpha, as might be expected from their individual horizon-dependence. Instead, co-skewness has an inherent sensitivity to the investment horizon, but without a frictionless market the estimation of co-skewness may be perturbed by autocorrelation and intertemporal cross-correlations.

3. Data and Empirical Facts

3.1. Data

To examine empirically the horizon effect on co-skewness, our initial sample consists of all NYSE/AMEX/NASDAQ-listed stocks with available data from the Center for Research in Security Prices (CRSP). Alongside prices, the number of outstanding shares, trading volumes and adjusted dividends are also collected. We use stocks with share codes 10 or 11. We include both listed and dead firms. Thus, our data set is free of any potential survivorship bias. Our sample period is August 1, 1962 to December 31, 2020.

There is no agreement on the horizons to be examined among studies on the horizon effect. Kamara et al. (2016) and Brennan and Zhang (2019) consider longer horizons from one month to 60 months, while studies such as Hawawini (1980a) and Corhay (1992) use shorter horizons of less than a month. We follow the choice of Handa et al. (1989), where both short and long horizons are considered. Nine horizons, in total, are considered. Daily, bi-daily, weekly and bi-weekly (1-day, 2-day, 5-day and 10-day) logarithmic returns are collected or constructed from the CRSP daily tape, and returns with longer horizons (1-month to 12-months) are from the CRSP monthly tape. For all horizons, we primarily employ self-calculated value-weighted returns using all stocks as the market index.[7] The risk-free rate is the (compounded) three- month T-bill rate, and we also scale the rate for each sampling horizon.

Following previous research examining the horizon effect, we detail our empirical results by forming portfolios based on the market capitalization of securities. As mentioned above, Theobald and Yallup (2004) suggest that the speed of pricing adjustment for larger firms is greater than that for smaller counterparts. Post et al. (2008) also examines market capitalization-based portfolios, indicating that gamma decreases from small to larger capitalization stocks. Therefore, we hypothesize that

[7] We do not use the CRSP index as our market index because its returns are not logarithmic, and therefore are not additive. Our daily market returns are reasonable and closely track the daily CRSP index having a correlation coefficient of 0.96.

larger firms have a relatively smaller horizon effect on co-skewness. We allocate all securities into 10 deciles based on the NYSE break- points (NYSE breakpoints are collected from the Kenneth French data library).

Table 1 documents the intertemporal cross-correlations between each size decile and the market portfolio. The "1-lag" ("1-lead") cross-correlation coefficient indicates the cross-correlation between the decile returns and market returns lagged by (leading by) one period. For all size deciles, we observe an increase in 1-lag serial cross-correlations and autocorrelation from daily data to monthly data. This increase in serial cross-correlation will impact upon the estimation of co-skewness, as demonstrated in our discussions on the long-run horizon effect in Section 2.4. Moreover, returns of smaller firms have greater cross-correlations to market returns. This supports the evidence detailed by previous literature that the speed of pricing adjustment for larger firms is relatively more rapid. Finally, we observe some evidence for significant serial cross-correlation and autocorrelation for daily data for a lag of 5 days, highlighting how pervasive autocorrelation in returns is.

TABLE 1 ABOUT HERE

3.2. Sensitivity of Estimated Co-skewness to the Investment Horizon

In this subsection, we provide empirical evidence for the existence of a horizon effect on portfolio co-skewness. Co-skewness is calculated for each portfolio using a 15-year moving window, across nine horizons from 1 business day to 1 year. In each case, co-skewness is estimated relative to the value-weighted market index. Table 2 presents the averaged co-skewness using horizons of 1, 2, 5, and 10 days, and 1 to 12 months, and details associated test statis- tics. Figure 1 illustrates the magnitude of co-skewness averaged for three representative deciles (Decile 1, Decile 6 and Decile 10) along with associated confidence intervals.

We document the average co-skewness for each size decile across horizons, showing clear evidence of an horizon effect on co-skewness. This horizon effect is particularly evident for deciles comprising the smallest and largest capitalization stocks. For example, at a 1-day horizon co-skewness for Decile 1 is -0.520, increasing to 0.097 and 0.124 for 6-month and 12-month horizons, respectively. This also highlights a second novel finding. For the first four size deciles, the sign of co-skewness reverses as we move from short to long-horizons. While Deciles 8 and 9 display negative co-skewness at all horizons, Decile 10 has positive co-skewness for all but the 6 and 12 -month horizon, where co-skewness is found to be only marginally negative (-0.008 or -0.004). These findings relate to previous work, which demonstrates that betas of smaller firms tend to increase, while those for the largest firms decline as the investment horizon lengthens (Cohen et al., 1983b).

One possible explanation for the empirical phenomena described relates to the impact of price delay. Given that firm size is a proxy for price adjustment delay, the magnitude of estimated co-skewness may be influenced by the speed of stock price reaction to market wide information. In keeping with this notion, co-skewness estimates for portfolios consisting of stocks with smaller capitalization display strong variation from the shortest to longest horizons.

To better examine the relation between the strength of the horizon effect and the price delay, Panel B summarizes three measures through which we capture the relative size and significance of the horizon effect in co-skewness. The extant literature considering the horizon effect has not specified a standard test to measure the strength of the horizon effect. The standard deviations of estimates for the same security over horizons and the F-statistics of ANOVA1 are suggested as indicators (Corhay, 1992). As our second measure, we augment the use of ANOVA1, with a One-Way Repeated Measures ANOVA, since tests of significance for ANOVA1 are known to be valid only if the samples are independent and the variance of each sample is equal.[8] Therefore, in our tests, if there is no horizon effect, the standard deviation and augmented F-statistics should be zero, while larger values indicate a stronger effect. Furthermore, we define an intuitive variable, referred to as Estimation Bias or EB_(γ,i), as the sum of the absolute differences between other horizon estimates and monthly estimates:

$$EB_{\gamma,i} = \sum_n |\bar{\gamma}_{\mathrm{in,t}} - \bar{\gamma}_{\mathrm{i}T_m,t}| n \in \{thechoiceofhorizons\} \quad (6)$$

where $\bar{\gamma}_{\mathrm{in,t}}$ are the averaged co-skewness for an n - day horizon and T_m stands for the length of a business month. EB quantitatively captures the aggregate tendency of co-skewness estimates to vary from that measured at a monthly horizon, given monthly data is mostly employed in the existing literature. This measure of estimation bias is zero if there is no horizon effect.

We reject the null hypothesis that co-skewness is equal to that estimated at a monthly hori- zon at a 1% level for the three measures shown in Panel B. Specifically, for all securities with smaller market capitalization relative to that of the averaged market capitalizations of firms constituting the market portfolio, the magnitudes of the horizon effect on their co-skewness are inversely related to their market capitalizations. For example, Decile 8 has the smallest standard deviations of 0.0095 and estimation bias of 0.707, and Decile 9 has the lowest F- value. Companies in the first 9 deciles all have lower market capitalizations than the market average and their standard deviations are found to be inversely related to the market capitalization. In

[8] The one-way repeated measures ANOVA (also known as a within-subjects ANOVA) is ideal in testing the horizon effect, since it is used to determine whether three or more group means are different where the participants are the same in each group.

contrast, for Decile 10 in which firms have greater-than-average size, the sensitivity of co-skewness estimates to the investment horizon is considerably higher compared to those of Decile 8 and Decile 9. These findings may suggest that, for any security, the greater the deviation in the firm's size relative to the average size of companies in the market index, the more sensitive the co-moments will be to the selection of the investment horizon.

Furthermore, our findings for co-skewness sign reversal indicate that, in addition to price delay, co-skewness may be affected by other factors. If price delay is the sole driver of the horizon effect (as concluded by the literature describing the horizon effect on betas), the estimated co-skewness should converge to zero for longer horizons, as price delay timelines become smaller relative to the horizon considered. In contrast, we find that the estimated co-skewness of a portfolio tends to increase (i.e. Decile 1) or decrease (i.e. Decile 10) for longer horizons. This may be related to the clientele effect suggested by Kamara et al. (2016). Investors with different horizons may have different preferences on skewness, given that the risk aversion is also found to be horizon-dependent (Andries et al., 2019). Thus, the co-skewness of an asset or a portfolio that is preferred by shorter-horizon investors, as it has negative co-skewness, may be positive at longer horizons.

Given our novel findings documented, in the section which follows, we provide theoretical evidence that, while price delay is a leading reason for the for the horizon effect on co-skewness reported, it may not be the only one. Moreover, we examine the different pricing roles of co- skewness risk at short, intermediate and long horizons.

3.3. Model of Horizon-Dependent Co-skewness

We now test if the theoretical decomposition, Equation 3, of co-skewness using short- horizon returns while accounting for intertemporal cross-covariance provides an adequate de- scription of the properties of empirical co-skewness estimates. In Table 3, we illustrate the estimates of co-skewness originating from our horizon-dependent model for Decile 1 and Decile 10. The focus is on unconditional co-skewness estimates using data over the period 1962 - 2020.

TABLE 3 ABOUT HERE

To calibrate Equation 3, we require estimates of residual returns $\varepsilon_{iT,t}$ and $\varepsilon_{mT,t}$ at the shortest horizon under consideration (1-Day). Equation 3 also requires estimates of long-horizon beta. Beta can, in turn, be either empirically estimated using long horizon data or model-generated from short-horizon data using Equation 4. In Table 3, beta is empirically estimated using long horizon data, but results are consistent for model generated betas.

Panel A of Table 3 details the model estimate of co-skewness by incorporating different orders of serial-covariance and intertemporal cross-covariance. The first row lists the simulated results without any serial covariance terms (0 order). Moving from short to long horizons, a clear monotonic increase in co-skewness is observed, with the long horizon theoretical estimate converging towards zero. These findings provide support for the $1/\sqrt{T}$ rule, described in Proposition 2. In the absence of serial-covariance and intertemporal cross-covariance, co-skewness estimates converge towards zero. The results, however, present a clear difference be- tween modelled co-skewness with no serial covariance and estimates found using long-horizon data. For example, for Decile 1 the model suggests a value of -0.057 for 3-month (65-day) co-skewness, while the empirical estimate is 0.246. These differences can be attributed to a presence of intertemporal covariance terms which, in the sense of Hawawini (1983), bias the estimation of co-skewness.

Including intertemporal covariance terms of order 1, we see that the 5-Day theoretical estimate of -0.418 is now approximately in keeping with that of the 5-Day empirical estimate. Similar findings are evident at higher orders. At the limit, we consider up to 64 intertemporal covariance terms and we observe that 3-month theoretical co-skewness increases from -0.057 to 0.242 as the number of intertemporal covariance terms is increased, where the latter is in keeping with the estimate from long horizon data. Similar findings are evident for the largest decile. This highlights the importance of intertemporal covariance terms in the estimation of long-horizon co-skewness. Furthermore, these results illustrate why empirical co-skewness estimates do not follow the $1/\sqrt{T}$ rule described in Proposition 2, as they are perturbed by intertemporal covariance in the underlying time series from the theoretical level where such terms are absent.

In panel B, we also examine the importance of intertemporal relations in the underlying time-series on co-skewness estimates at different horizons. The objective here is to determine the extent of the impact of serial- and cross-serial correlation on co-skewness. To this end, we add increments to the serial- and cross-serial correlation terms and examine whether this changes the magnitude and sign of co-skewness. 1st and 4th order intertemporal relations are considered as examples. Our findings indicate that adding larger increments in absolute value terms (e.g. +0.03 or -0.03) further biases the estimation of co-skewness.

For example, considering just 1st order intertemporal terms, the estimate of co-skewness from the model in Panel A is -0.418 at a 5-Day horizon. Including an increment of -0.03 (0.03) to the 1st order serial correlation and cross-correlation terms, we see that the magnitude of co- skewness changes to -0.298 (-0.133). Highlighting the importance of higher order terms, when the same increment is applied to the first four serial and cross-correlation terms, co-skewness ranges from -

0.576 to -0.316, bracketing the estimated co-skewness level. Similarly, for Decile 10 with 4th order intertemporal terms, co-skewness is 0.334 using 5-Day data. Incrementing both serial and cross-correlations by amounts ranging from -0.03 to 0.03 results in co-skewness estimates ranging from 0.046 to 0.388.

Our model of co-skewness horizon dependence shows a clear relationship with intertemporal correlation terms. Without such terms the scaling law derived in Equation 5 holds, with co-skewness converging to zero at long horizons. For example, in the first row where zero-order serial correlation and cross-correlation are considered, the 3-Month estimate (-0.057) is approximately $1/\sqrt{65}$ of the 1-Day co-skewness (-0.457). In other words, co-skewness has an inherent horizon dependence, but this may be masked in long horizon co-skewness estimates due to intertemporal covariance. Incorporating various lags of interdependence we provide evidence that co-skewness estimates at different horizons are impacted by underlying time series characteristics. These findings, along with the earlier empirical results surrounding co-skewness estimates, may have implications for higher-order asset pricing, a question we address next.

4. Higher-order Asset Pricing and Horizon-Dependent Co-skewness

We next explore implications for higher-order asset pricing. Harvey and Siddique (2000) and Kostakis et al. (2012), among other studies, constructed a co-skewness risk factor, CSK, in a similar fashion to the Fama and French (1993) factors by using return spreads between portfolios with the lowest 30% and highest 30% co-skewness. In this paper we reconstruct the co-skewness factor by estimating co-skewness at each horizon and sorting stocks into co- skewness deciles. The descriptive statistics for each co-skewness decile, as well as return spreads between co-skewness Decile 1 and Decile 10 along with the constructed CSK pricing factor, are shown in Table 4.

TABLE 4 ABOUT HERE

We find evidence of significant variation in co-skewness across the 10 deciles, demonstrating that co-skewness is a meaningful sorting criterion across all horizons. Kostakis et al. (2012) suggest that investors require a premium to hold shares with negatively co-skewed returns, which indicates a lower return for shares with higher co-skewness. Therefore, the decile with the most negatively co-skewed shares (Dec1) should yield a significantly higher average excess return relative to Dec10, if the co-skewness factor can capture the cross-sectional expected re- turns. We find this to be the case for co-skewness deciles using returns with 5-Day, 10-Day and 1-Month horizons. For each of these there is a significant return spread between Decile 1 and Decile 10. Consistent results are found for the co-skewness factor, CSK. Therefore, we can hypothesize that the pricing roles of the co-skewness factor (CSK) might be only significant when intermediate horizons (5-Day, 10-Day and 1-Month) horizons are considered. At long horizons, we provide evidence that the returns on deciles increase as we move from Decile 1 to Decile 10. These contrasting results between short- and long-horizons, along with the evidence for co-skewness estimates converging towards zero at long horizons provided above, indicate that the risk premium associated with co-skewness may not persist at long-horizons. We next investigate this formally.

Next, using the Fama and MacBeth (1973) two-step method at different investment horizons, we assess whether the cross-sectional variation in asset returns can be explained by expo- sure to co-skewness. In order to compare the results across horizons, we follow the approach of Kamara et al. (2016). In the first stage, we run time-series regressions for each individual stock to estimate the moving window (15-year) k-month (or -day) factor exposure. For example, the 15-year 6-month exposures of CSK (β_6^{CSK}) factors in the month t are estimated using overlapping 6-month excess 6returns and the 6-month co-skewness factor using the past 15-year window. β_6^{CSK} stands for the exposure of the k-month co-kurtosis factor which is defined as the k-month spread returns of securities with 30% largest co-kurtosis and those with 30% lowest, analogous to those presented for CSK. For all co-skewness betas in the regression of month t, the average beta of the firms in the co-skewness decile is used for the firm beta.

Then, in the second-stage, in addition to including β_k^{MKT}, β_k^{CSK} and β_k^{CKT} for k-month (or-day) horizons, we also control for the variable SIZE that is the natural logarithm of market capitalization measured at the end of month t-1 and the variable B/M is the book-to-market ratio of month t. We use the book value of the fiscal year ending in year y-1 and market value in December of year y-1 for the 12 months from July of year y to June of year y+1. For longer horizons formed using monthly returns, we also include several more commonly used covariates, such as the momentum and the short-term price reversal factors as used in Kamara et al. (2016), as well as long-term reversal (McLean, 2010), illiquidity ratios (Amihud, 2002) and idiosyncratic volatility (Ang et al., 2006) factors.[9]

Table 5, Panel (a) reports results for the Fama-MacBeth second stage regressions. The slope coefficients are generated as average values across all months. The t-statistics (in brackets) of the cross-sectional regression coefficients are calculated using Newey-West standard errors to reduce the impact of overlapping returns.

TABLE 5 ABOUT HERE

Results provide further support for the idea that the implications of stock co-

[9] We calculate the momentum (R12, 2) as the 11-month cumulative return in months [t-12, t-2], the short-term price reversal (R1, 1) as the return in [t, t-1] while the long-term as (R60, 13). The illiquidity measure follows the set-up of Amihud (2002) and monthly idiosyncratic volatility is calculated from daily residual terms in each month of basic CAPM model as suggested by Ang et al. (2006). These factors are examined in the literature where monthly or longer-horizon returns are mainly considered. Thus, we do not add them when daily and other shorter-horizons are tested.

skewness are different dependent on the horizon examined. Specifically, the co-skewness factor, CSK, is found to only be priced at 2-day, 5-day, 10-day and 1-month horizons. For horizons longer than 1 month, there is no evidence that CSK is a priced factor.[10] When we control for other variables our findings are unchanged. CKT, the co-kurtosis factor, is found to have a significant negative price of risk at a 10 day and 1 month horizon. Size, momentum and reversal are related to excess returns at all horizons. The book to market variable, B/M, is not found to be linked with excess returns. Results for two subsamples for the period 1962-1989 and 1990-2020 are shown in Table 5, Panels (b) and (c) respectively. Findings are consistent, with co-skewness found to be priced only at short horizons.

Our empirical evidence adds to the body of literature which indicates that previous findings relating to co-skewness result from the specific empirical set-up employed. In the sense of Post and Levy (2005) and Post et al. (2008), the results documented here might be interpreted as evidence against the co-skewness premium as providing support for the three-factor model. If institutional investors have long-run investment horizons, then our findings indicate that the co-skewness premium is not obtainable for such investors. Assessing the Fama and French (1993) factors, Kamara et al. (2016) propose that clientele effects result in the appropriate horizon to assess risk factors differing across factors. In the context of the results detailed here, this might be a consequence of short run investors being compensated for exposures to shocks to which long-run investors are less sensitive.

5. Discussion and Concluding Remarks

Co-skewness extends the Sharpe-Lintner CAPM, allowing for more detailed characterization of individual asset risk. While the extant literature has highlighted the importance of co-skewness in explaining the cross-section of stock returns, the choice and implications of the return horizon selected have not been considered. Based on an extensive data sample of stocks from the CRSP database for the period 1962 to 2020, this paper details the sensitivity of co-skewness estimates to the return horizon. Developing a system of consistent sequential tests, we also measure the strength of the horizon effect for co-skewness estimation. In order to develop an understanding as to the drivers of horizon-dependent co-skewness, we propose a model of long-horizon co-skewness which employs only data at the shortest horizon. Using this, we provide evidence that serial- and intertemporal cross-correlations influence the estimation of co-skewness, but that co-skewness also has an inherent horizon dependence. Finally, we assess whether co-skewness is priced at different horizons.

Accordingly, this study sheds new light on research considering higher-order asset

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[10] In other regressions, examining only the relationship between excess stock returns and two factors, MKT and CSK, identical results were obtained, indicating that our findings are not a consequence of the inclusion of other control variables.

pricing and portfolio selection. First, we provide evidence that the magnitudes of the estimates of third moment pricing coefficients (gamma) are significantly influenced by sampling horizon. More- over, the sign of gamma may reverse for longer horizons. Second, we refine our estimation of gammas by using returns of market-capitalization sorted portfolios measured over horizons from one day to twelve months. Firm market capitalization is used as a proxy for price adjustment delay. We provide evidence that the horizon effect on co-skewness is strongest for the smallest and largest companies, in keeping with the premise of price adjustment delays. We also propose a “scaling law” for co-skewness, highlighting an inherent horizon dependency for co-skewness. Finally, we examine implications for asset pricing and show that co-skewness is only priced at short horizons ranging from two days to one month.

We conclude with a word of caution to empirical researchers who use higher-order asset pricing or portfolio selection in their empirical work. First, the literature documents some return anomalies such as the Size Effect (Banz, 1981) and Liquidity Effect (Amihud, 2002). Investors may purchase stocks with smaller capitalizations or less liquidity while seeking out opportunities for extra returns. Our results suggest that investors in smaller or less liquid firms should pay greater attention to the choice of the investment horizon in analysing the higher-order systematic risk exposure of such securities. Second, recent portfolio selection theory suggests that risk-averse investors are attracted to securities with positive co-skewness estimates. One implication of our findings, however, is that the relative ranks of portfolio co-skewness alter when the sampling horizon is lengthened. In particular, co-skewness is significantly sensitive to the investment horizon, since not only the magnitudes but also the signs of co-skewness change. An asset that is selected based on its positive co-skewness using a particular sampling horizon may have negative co-skewness using another horizon. To sum up, the horizon effect presents potential ambiguity in pricing and the selection of assets for different situations.

References

A¨ıt-Sahalia, Y. and Brandt, M. W. (2001), 'Variable selection for portfolio choice', *The Journal of Finance* 56(4), 1297–1351.

Amihud, Y. (2002), 'Illiquidity and stock returns: cross-section and time-series effects', *Journal of Financial Markets* 5(1), 31–56.

Andries, M., Eisenbach, T. M. and Schmalz, M. C. (2019), 'Horizon-dependent risk aversion and the timing and pricing of uncertainty', *Working Paper Available at SSRN 2535919*.

Ang, A., Hodrick, R. J., Xing, Y. and Zhang, X. (2006), 'The cross-section of volatility and expected returns', *The journal of finance* 61(1), 259–299.

Bandi, F. M., Chaudhuri, S. E., Lo, A. W. and Tamoni, A. (2021), 'Spectral factor models', Journal of Financial Economics.

Banz, R. W. (1981), 'The relationship between return and market value of common stocks', *Journal of Financial Economics* 9(1), 3–18.

Barun´ık, J. and Kehl´ık, T. (2018), 'Measuring the frequency dynamics of financial connected- ness and systemic risk', *Journal of Financial Econometrics* 16(2), 271—-296.

Benartzi, S. and Thaler, R. H. (1995), 'Myopic loss aversion and the equity premium puzzle', *The Quarterly Journal of Economics* 110(1), 73–92.

Boguth, O., Carlson, M., Fisher, A. J. and Simutin, M. (2016), 'Horizon effects in average re- turns: The role of slow information diffusion', *The Review of Financial Studies* 29(8), 2241– 2281.

Boudt, K., Cornilly, D. and Verdonck, T. (2020), 'A coskewness shrinkage approach for estimating the skewness of linear combinations of random variables', *Journal of Financial Econometrics* 18(1), 1–23.

Brennan, M. J., Jegadeesh, N. and Swaminathan, B. (1993), 'Investment analysis and the adjustment of stock prices to common information', *Review of Financial Studies* 6(4), 799–824.

Brennan, M. and Zhang, Y. (2019), 'Capital asset pricing with a stochastic horizon', *Journal of Financial and Quantitative Analysis,* 55(3), 783–827.

Cohen, K. J., Hawawini, G. A., Maier, S. F., Schwartz, R. A. and Whitcomb, D. K. (1980), 'Implications of microstructure theory for empirical research on stock price behavior', *The Journal of Finance* 35(2), 249–257.

Cohen, K. J., Hawawini, G. A., Maier, S. F., Schwartz, R. A. and Whitcomb, D. K. (1983*a*), 'Estimating and adjusting for the intervalling-effect bias in beta', *Management Science* 29(1), 135–148.

Cohen, K. J., Hawawini, G. A., Maier, S. F., Schwartz, R. A. and Whitcomb, D. K. (1983*b*), 'Friction in the trading process and the estimation of systematic risk', *Journal of Financial Economics* 12(2), 263–278.

Cohen, L. and Frazzini, A. (2008), 'Economic links and predictable returns', *The Journal of Finance* 63(4), 1977–2011.

Conlon, T., Cotter, J. and Gencay, R. (2018), 'Long-run wavelet-based correlation for financial time series', *European Journal of Operational Research* 271(2), 676–696.

Conrad, J., Dittmar, R. F. and Ghysels, E. (2013), 'Ex ante skewness and expected stock re- turns', *The Journal of Finance* 68(1), 85–124.

Corhay, A. (1992), 'The intervalling effect bias in beta: A note', *Journal of Banking and Finance* 16(1), 61–73.

Dierkes, M., Erner, C. and Zeisberger, S. (2010), 'Investment horizon and the attractiveness of investment strategies: A behavioral approach', *Journal of Banking & Finance* 34(5), 1032– 1046.

Dimson, E. (1979), 'Risk measurement when shares are subject to infrequent trading', *Journal of Financial Economics* 7(2), 197–226.

Dittmar, R. F. (2002), 'Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns', *The Journal of Finance* 57(1), 369–403.

Eberl, A. and Klar, B. (2020), 'Asymptotic distributions and performance of empirical skewness measures', *Computational Statistics & Data Analysis* 146, 106939.

Engle, R. F. (2011), 'Long-term skewness and systemic risk', *Journal of Financial Econometrics* 9(3), 437–468.

Fama, E. F. and French, K. R. (1993), 'Common risk factors in the returns on stocks and bonds', *Journal of Financial Economics* 33(1), 3–56.

Fama, E. F. and French, K. R. (2018), 'Long-horizon returns', *The Review of Asset Pricing Studies* 8(2), 232–252.

Fama, E. F. and MacBeth, J. D. (1973), 'Risk, return, and equilibrium: Empirical tests', *The Journal of Political Economy* pp. 607–636.

Fang, H. and Lai, T.-Y. (1997), 'Co-kurtosis and capital asset pricing', *Financial Review* 32(2), 293–307.

Genc¸ay, R., Selc¸uk, F. and Whitcher, B. (2005), 'Multiscale systematic risk', *Journal of Inter- national Money and Finance* 24(1), 55–70.

Guidolin, M. and Timmermann, A. (2006), 'Term structure of risk under alternative econometric specifications', *Journal of Econometrics* 131(1-2), 285–308.

Guidolin, M. and Timmermann, A. (2008), 'International asset allocation under regime switch- ing, skew, and kurtosis preferences', *The Review of Financial Studies* 21(2), 889–935.

Handa, P., Kothari, S. and Wasley, C. (1989), 'The relation between the return interval and betas: Implications for the size effect', *Journal of Financial Economics* 23(1), 79–100.

Harvey, C. R. and Siddique, A. (2000), 'Conditional skewness in asset pricing tests', *The Journal of Finance* 55(3), 1263–1295.

Hawawini, G. (1983), 'Why beta shifts as the return interval changes', *Financial Analysts Journal* 39(3), 73–77.

Hawawini, G. A. (1980*a*), 'An analytical examination of the intervaling effect on skewness and other moments', *Journal of Financial and Quantitative Analysis* 15(5), 1121–1127.

Hawawini, G. A. (1980*b*), 'Intertemporal cross-dependence in securities daily returns and the short-run intervaling effect on systematic risk', *Journal of Financial and Quantitative Anal- ysis* 15(1), 139–149.

Hou, K. and Moskowitz, T. J. (2005), 'Market frictions, price delay, and the cross-section of expected returns', *Review of Financial Studies* 18(3), 981–1020.

Hur, J. and Singh, V. (2017), 'Cross-section of expected returns and extreme returns: The role of investor attention and risk preferences', *Financial Management* 46(2), 409–431.

Jacquier, E. and Okou, C. (2014), 'Disentangling continuous volatility from jumps in long-run risk–return relationships', *Journal of Financial Econometrics* 12(3), 544—-583.

Jegadeesh, N. and Titman, S. (1995), 'Overreaction, delayed reaction, and contrarian profits', *Review of Financial Studies* 8(4), 973–993.

Jondeau, E. and Rockinger, M. (2012), 'On the importance of time variability in higher moments for asset allocation', *Journal of Financial Econometrics* 10(1), 84–123.

Kalev, P. S., Saxena, K. and Zolotoy, L. (2019), 'Coskewness risk decomposition, covariation risk, and intertemporal asset pricing', *Journal of Financial and Quantitative Analysis* 54(1), 335–368.

Kamara, A., Korajczyk, R. A., Lou, X. and Sadka, R. (2016), 'Horizon pricing', *Journal of Financial and Quantitative Analysis* 51(6), 1769–1793.

Kim, T.-H. and White, H. (2004), 'On more robust estimation of skewness and kurtosis', *Finance Research Letters* 1(1), 56–73.

Kostakis, A., Muhammad, K. and Siganos, A. (2012), 'Higher co-moments and asset pricing on london stock exchange', *Journal of Banking and Finance* 36(3), 913 – 922.

Kraus, A. and Litzenberger, R. (1983), 'On the distributional conditions for a consumption- oriented three moment capm', *The Journal of Finance* 38(5), 1381–1391.

Kraus, A. and Litzenberger, R. H. (1976), 'Skewness preference and the valuation of risk as- sets', *The Journal of Finance* 31(4), 1085–1100.

Lambert, M. and Hu¨ bner, G. (2013), 'Comoment risk and stock returns', *Journal of Empirical Finance* 23(0), 191 – 205.

Langlois, H. (2020), 'Measuring skewness premia', *Journal of Financial Economics* 135(2), 399 – 424.

Lau, H.-S. and Wingender, J. R. (1989), 'The analytics of the intervaling effect on skewness and kurtosis of stock returns', *Financial Review* 24(2), 215–233.

Lau, H.-S., Wingender, J. R. and Lau, A. H.-L. (1989), 'On estimating skewness in stock returns', *Management Science* 35(9), 1139–1142.

Levhari, D. and Levy, H. (1977), 'The capital asset pricing model and the investment horizon', *The Review of Economics and Statistics* 59(1), 92–104.

Lintner, J. (1965), 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *The Review of Economics and Statistics* pp. 13–37.

Lo, A. W. and MacKinlay, A. C. (1990), 'When are contrarian profits due to stock market overreaction?', *Review of Financial Studies* 3(2), 175–205.

Martellini, L. and Ziemann, V. (2010), 'Improved estimates of higher-order comoments and implications for portfolio selection', *Review of Financial Studies* 23(4), 1467–1502.

McLean, R. D. (2010), 'Idiosyncratic risk, long-term reversal, and momentum', *Journal of Financial and Quantitative Analysis* 45(4), 883–906.

Mitton, T. and Vorkink, K. (2007), 'Equilibrium underdiversification and the preference for skewness', *Review of Financial Studies* 20(4), 1255–1288.

Neuhierl, A. and Varneskov, R. T. (2021), 'Frequency dependent risk', *Journal of Financial Economics* 140(2), 644–675.

Patton, A. J. (2004), 'On the out-of-sample importance of skewness and asymmetric dependence for asset allocation', *Journal of Financial Econometrics*

2(1), 130–168.

Perron, P., Chun, S. and Vodounou, C. (2013), 'Sampling interval and estimated betas: Implications for the presence of transitory components in stock prices', *Journal of Empirical Finance* 20, 42–62.

Post, T. and Kopa, M. (2017), 'Portfolio choice based on third-degree stochastic dominance', *Management Science* 63(10), 3381–3392.

Post, T. and Levy, H. (2005), 'Does risk seeking drive stock prices? A stochastic dominance analysis of aggregate investor preferences and beliefs', *The Review of Financial Studies* 18(3), 925–953.

Post, T., Van Vliet, P. and Levy, H. (2008), 'Risk aversion and skewness preference', *Journal of Banking & Finance* 32(7), 1178–1187.

Poti, V. and Wang, D. (2010), 'The coskewness puzzle', *Journal of Banking and Finance* 34(8), 1827–1838.

Rubinstein, M. E. (1973), 'The fundamental theorem of parameter-preference security valuation', *Journal of Financial and Quantitative Analysis* 8(1), 61–69.

Scholes, M. and Williams, J. (1977), 'Estimating betas from nonsynchronous data', *Journal of Financial Economics* 5(3), 309–327.

Sharpe, W. F. (1964), 'Capital asset prices: A theory of market equilibrium under conditions of risk', *The Journal of Finance* 19(3), 425–442.

Smith, D. R. (2007), 'Conditional coskewness and asset pricing', *Journal of Empirical Finance* 14(1), 91–119.

Theobald, M. and Yallup, P. (2004), 'Determining security speed of adjustment coefficients', *Journal of Financial Markets* 7(1), 75–96.

Wang, J.-N., Yeh, J.-H. and Cheng, N.-P. (2011), 'How accurate is the square-root-of-time rule in scaling tail risk: A global study', *Journal of Banking & Finance* 35(5), 1158–1169.

Zhang, X. (2006), 'Information uncertainty and stock returns', *The Journal of Finance* 61(1), 105–137.

Table 1: Intertemporal Correlation Tests among Size Deciles and the Market Index across Horizons

		Panel A: Intertemporal correlations using daily returns					
Size Decile	Mkt. Cap.	Cross-correlations				Auto-correlations	
	(Million $)	1-lag	5-lag	1-lead	5-lead	1-lag	5-lag
Decile 1	42.710	0.146***	0.055***	−0.012	−0.007	0.148***	0.070***
Decile 2	113.237	0.071***	0.031***	−0.007	−0.007	0.042***	0.036***
Decile 3	200.913	0.076***	0.025***	0.001	−0.007	0.058***	0.025***
Decile 4	318.976	0.079***	0.016**	0.003	−0.003	0.061***	0.019**
Decile 5	485.273	0.082***	0.015*	0.011	−0.006	0.072***	0.013**
Decile 6	723.657	0.090***	0.014*	0.013	−0.003	0.084***	0.015*
Decile 7	1114.856	0.093***	0.010	0.020**	−0.007	0.094***	0.007
Decile 8	1895.457	0.075***	0.005	0.019**	−0.005	0.075***	0.003
Decile 9	3842.743	0.043***	0.002	0.019**	−0.002	0.045***	0.003
Decile 10	39384.100	−0.020**	−0.010	0.022***	0.000	−0.015*	−0.007

		Panel B: Intertemporal correlations using monthly returns					
	Mkt. Cap.	Cross-correlations				Auto-correlations	
	(Million $)	1-lag	5-lag	1-lead	5-lead	1-lag	5-lag
Decile 1	42.710	0.209***	0.040	0.098***	0.047	0.184***	−0.014
Decile 2	113.237	0.179***	0.045	0.100***	0.040	0.159***	−0.003
Decile 3	200.913	0.176***	0.043	0.113***	0.050*	0.172***	0.006
Decile 4	318.976	0.148***	0.049*	0.110***	0.048	0.138***	0.017
Decile 5	485.273	0.145***	0.066**	0.111***	0.041	0.144***	0.029
Decile 6	723.657	0.134***	0.068**	0.108***	0.055*	0.134***	0.049
Decile 7	1114.856	0.116***	0.063**	0.121***	0.048	0.130***	0.038
Decile 8	1895.457	0.097***	0.074**	0.118***	0.059**	0.104***	0.059**
Decile 9	3842.743	0.099***	0.079***	0.109***	0.065**	0.100***	0.071**
Decile 10	39384.100	0.094***	0.080***	0.102***	0.085***	0.089***	0.090***

Note: In Panel A, stocks from August, 1962 to December, 2020 on the CRSP daily tape are used. Stocks are allocated into 10 size deciles based on the NYSE breakpoints. The first, Decile 1, contains the smallest firms and the last, Decile 10, contains the largest. For each size portfolio, the intertemporal cross-correlations between the lead and lag market portfolio and size portfolios are also calculated. The "1-lag" cross-correlation coefficient indicates the cross-correlation between the size portfolio returns and 1-lag market returns. ***, ** and * indicate that we reject the null hypothesis that there is no serial correlations at 99%, 95% and 90% level, respectively. Panel B follows the same estimation process by using stocks from August, 1962 to December, 2020 on the CRSP monthly tape.

Table 2: Co-skewness of Size Deciles across Horizons

Size Decile	Mkt. Cap (Million $)	Panel A: Mean co-skewness of each decile									Panel B: Measures of estimation variation		
		1D	2D	5D	10D	1M	2M	3M	6M	12M	Standard Deviation	Repeated Measures ANOVA	Estimation Bias
Decile 1	42.710	−0.520	−0.477	−0.389	−0.325	−0.287	−0.132	−0.027	0.097	0.124	0.241***	55.042***	1.772***
Decile 2	113.237	−0.291	−0.420	−0.313	−0.313	−0.308	−0.112	−0.002	0.073	0.106	0.195***	32.335***	1.434***
Decile 3	200.913	−0.245	−0.363	−0.262	−0.258	−0.273	−0.082	0.003	0.079	0.113	0.174***	30.269***	1.349***
Decile 4	318.976	−0.258	−0.337	−0.241	−0.249	−0.252	−0.085	−0.001	0.070	0.097	0.162***	29.883***	1.196***
Decile 5	485.273	−0.192	−0.318	−0.213	−0.259	−0.236	−0.118	−0.072	−0.046	0.017	0.111***	11.503***	0.898***
Decile 6	723.657	−0.216	−0.325	−0.227	−0.219	−0.250	−0.089	−0.008	0.057	0.032	0.139***	22.832***	1.156***
Decile 7	1114.856	−0.263	−0.357	−0.266	−0.272	−0.190	−0.118	−0.050	0.015	0.007	0.135***	28.709***	1.013***
Decile 8	1895.457	−0.185	−0.269	−0.208	−0.203	−0.130	−0.081	−0.071	−0.005	−0.001	0.095***	9.167***	0.707***
Decile 9	3842.743	−0.295	−0.301	−0.255	−0.213	−0.047	−0.068	−0.110	−0.035	−0.039	0.114***	6.096***	0.980***
Decile 10	39384.100	0.259	0.327	0.263	0.271	0.225	0.131	0.089	−0.008	−0.004	0.125***	16.525***	0.911***

Note: Co-skewness estimated using value-weighted portfolio returns and the market index are illustrated in Panel A. Stocks are allocated into 10 size deciles based on the NYSE breakpoints. Daily returns of stocks from August, 1962 to December, 2020 on the CRSP daily tape are used when estimating 1-day, 2-day, 5-day and 10-day co-skewness, and similarly monthly returns of stocks on the CRSP monthly tape are used to estimate 1-month and longer-horizon co-skewness. The first, Decile 1, contains the smallest firms and the last, Decile 10, contains the largest. For each size decile and each horizon, we compute the co-skewness estimate at each month from August 1977 to December 2020 using the previous 15-year periods. This table details the average across all portfolio co-skewness estimates. In Panel B, the standard deviation, repeated measure ANOVA tests and estimation bias estimates, as in Equation 6 are used in detecting the magnitude of the horizon effect.

***, ** and * indicate that the null hypothesis that the estimates are equal to zero is rejected at the 99%, 95% or 90% level, respectively.

Table 3: Modelled Co-skewness Estimates

Panel A: Modeling Co-skewness with corrections in different degrees of orders

	Section A: Simulated results for smallest-size portfolios (decile 1)							Section B: simulated results for largest-size portfolio (decile 10)						
Orders	1D	2D	5D	10D	1M	2M	3M	1D	2D	5D	10D	1M	2M	3M
0	−0.457	−0.323	−0.204	−0.144	−0.102	−0.072	−0.057	0.306	0.216	0.137	0.097	0.068	0.048	0.038
1		−0.430	−0.204	−0.128	−0.085	−0.058	−0.045		0.269	0.250	0.166	0.057	0.044	0.037
4			−0.418	−0.424	−0.335	−0.246	−0.199			0.334	0.292	0.283	0.226	0.186
9				−0.398	−0.214	−0.106	−0.082				0.220	0.183	0.112	0.094
19					−0.142	0.129	0.181					0.157	0.091	0.046
39						0.153	0.222						0.085	0.058
64							0.242							0.067
Estimated	−0.457	−0.431	−0.417	−0.398	−0.147	0.155	0.246	0.306	0.269	0.335	0.218	0.154	0.085	0.063

Panel B: Modeling co-skewness with corrections for different serial- and cross-serial correlation increments

Orders	Increments	2D	5D	10D	1M	2M	3M	Increments	2D	5D	10D	1M	2M	3M
1	−0.03	−0.555	−0.298	−0.193	−0.130	−0.089	−0.069	−0.03	0.095	0.158	0.160	0.129	0.096	0.076
	−0.02	−0.510	−0.264	−0.169	−0.113	−0.078	−0.060	−0.02	0.171	0.218	0.201	0.095	0.077	0.061
	−0.01	−0.469	−0.233	−0.148	−0.099	−0.067	−0.052	−0.01	0.294	0.382	0.211	0.068	0.051	0.048
	+0.01	−0.395	−0.178	−0.111	−0.073	−0.050	−0.039	+0.01	0.236	0.321	0.240	0.048	0.041	0.034
	+0.02	−0.361	−0.155	−0.095	−0.062	−0.043	−0.033	+0.02	0.155	0.206	0.173	0.031	0.028	0.036
	+0.03	−0.330	−0.133	−0.080	−0.052	−0.036	−0.028	+0.03	0.171	0.030	0.103	0.025	0.017	0.080
4	−0.03		−0.576	−0.464	−0.292	−0.183	−0.147	−0.03		0.046	0.058	0.061	0.051	0.039
	−0.02		−0.515	−0.452	−0.317	−0.216	−0.175	−0.02		0.252	0.193	0.196	0.160	0.117
	−0.01		−0.463	−0.438	−0.330	−0.236	−0.191	−0.01		0.305	0.265	0.276	0.200	0.164
	+0.01		−0.379	−0.409	−0.336	−0.251	−0.202	+0.01		0.343	0.343	0.321	0.233	0.193
	+0.02		−0.346	−0.395	−0.334	−0.252	−0.203	+0.02		0.319	0.456	0.520	0.307	0.225
	+0.03		−0.316	−0.382	−0.329	−0.251	−0.201	+0.03		0.388	0.641	0.589	0.458	0.363

Note: Co-skewness is modeled using data over the period August, 1962 to December, 2020. A value-weighted market index is constructed using the CRSP daily tape. Co-skewness is modelled using daily returns and estimates of intertemporal covariances. Results for the smallest-size decile (Decile 1) and largest-size decile (Decile 10) are illustrated as representative. Panel A details the modelled co-skewness for differing orders of intertemporal covariance. To match with the horizons used, the first row lists the simulated results if there is no serial or cross-serial covariance relationships (0 order), the second row when relationships up to 4-order are included, etc.. The row titled “Estimated” corresponds to the co-skewness estimated using returns data with the relevant horizons. The row named “Modeled” shows the modeled co-skewness taking into account all relevant intertemporal relationships. The row titled 1/ N highlights the expected co-skewness estimate in the absence of intertemporal relationships, in keeping with Proposition 2. Panel B further lists modelled results with fixed 1 or 4-order intertemporal relationships but with increments (Incre.) to the estimated serial- and cross-serial correlation of between -0.03 and 0.03.

Table 4: Performance and Characteristics of Deciles Constructed on the Basis of Co-skewness

Horizon		Dec1	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Dec10	Dec1 to Dec10	CSK
1D	Average Cosk.	−0.420	−0.233	−0.150	−0.095	−0.055	−0.018	0.022	0.070	0.134	0.269	−0.689***	0.006
	VW Returns	0.063	0.070	0.063	0.069	0.057	0.064	0.061	0.068	0.060	0.055	0.008	
2D	Average Cosk.	−0.360	−0.220	−0.154	−0.105	−0.065	−0.031	0.004	0.043	0.097	0.213	−0.573***	0.014**
	VW Returns	0.131	0.130	0.131	0.134	0.114	0.122	0.127	0.132	0.114	0.106	0.025***	
5D	Average Cosk.	−0.337	−0.197	−0.133	−0.086	−0.048	−0.011	0.027	0.068	0.122	0.242	−0.579***	0.033***
	VW Returns	0.335	0.320	0.295	0.312	0.313	0.299	0.313	0.287	0.285	0.263	0.072***	
10D	Average Cosk.	−0.435	−0.257	−0.175	−0.116	−0.067	−0.022	0.023	0.077	0.149	0.304	−0.739***	0.103***
	VW Returns	0.631	0.595	0.683	0.587	0.674	0.582	0.596	0.579	0.553	0.481	0.150***	
1M	Average Cosk.	−0.453	−0.289	−0.214	−0.158	−0.109	−0.061	−0.012	0.043	0.114	0.253	−0.706***	0.151**
	VW Returns	1.244	1.168	1.091	1.152	1.178	1.091	1.084	1.080	1.072	0.933	0.311***	
2M	Average Cosk.	−0.434	−0.268	−0.191	−0.132	−0.081	−0.032	0.018	0.073	0.145	0.292	−0.726***	−0.091
	VW Returns	2.069	2.098	2.287	2.246	2.313	2.134	2.159	2.233	2.028	2.001	0.067	
3M	Average Cosk.	−0.447	−0.269	−0.187	−0.122	−0.064	−0.009	0.047	0.109	0.188	0.352	−0.799***	−0.064
	VW Returns	3.079	3.179	3.265	3.418	3.521	3.270	3.318	3.010	3.299	3.106	−0.027	
6M	Average Cosk.	−0.464	−0.275	−0.183	−0.112	−0.049	0.010	0.072	0.140	0.227	0.403	−0.867***	−0.368
	VW Returns	5.762	6.325	6.324	6.494	6.742	6.402	5.883	6.397	6.393	6.394	−0.632	
12M	Average Cosk.	−0.436	−0.258	−0.164	−0.088	−0.020	0.047	0.113	0.185	0.276	0.445	−0.881***	−0.935
	VW Returns	11.873	12.175	13.433	12.590	12.545	12.846	12.912	13.137	13.239	13.331	−1.458*	

Note: This table reports the characteristics of co-skewness deciles during the period August 1962–December 2020. All shares on the CRSP daily tape and monthly tape are sorted at month t in ascending order according to their co-skewness values estimated via a rolling window of 15-year observations and they are assigned to 10 deciles. Dec1 is the decile containing the shares with the lowest (most negative) estimated co-skewness and Dec10 with the highest (most positive) co-skewness. The excess returns of these deciles are calculated at month t + 1 (i.e. post ranking returns). Dec1–Dec10 stands for the spread between decile 1 and decile 10. Deciles are re-balanced on a monthly basis. VW returns correspond to the returns of value weighted portfolios. CSK shows the values of the co-skewness factor constructed as the return on a portfolio taking a long position in stocks with co-skewness in the lowest 30% and a short position in stocks with co-skewness in the highest 30%. ***, ** and * indicate that the null hypothesis that the estimates are equal to zero is to rejected at 99%, 95% or 90% level, respectively.

Table 5: Fama-MacBeth Regression Results with a Co-skewness factor (CSK)

	Panel A: Full sample, CRSP 1962–2020								
	(1) 1D	(2) 2D	(3) 5D	(4) 10D	(5) 1M	(6) 2M	(7) 3M	(8) 6M	(9) 12M
MKT	−0.209***	−0.165**	−0.145**	−0.261***	−0.341***	−0.275***	−0.190***	−0.068	−0.032
	(−2.849)	(−2.401)	(−2.123)	(−3.690)	(−6.270)	(−4.908)	(−3.573)	(−1.580)	(−0.863)
CSK	0.206	0.172*	0.158*	0.223***	0.274***	0.081	−0.054	0.029	0.057
	(1.095)	(1.802)	(1.850)	(3.222)	(5.653)	(1.605)	(−0.917)	(0.596)	(1.094)
CKT	0.097	0.130*	0.029	−0.191***	−0.197***	0.091*	0.083*	−0.015	−0.008
	−1.364	−1.913	−0.338	(−2.767)	(−4.219)	(1.941)	(1.875)	(−0.282)	(−0.182)
B/M	−0.458***	−0.412***	−0.320***	−0.460***	0.210***	0.209***	0.135***	0.096***	0.092***
	(−4.331)	(−4.489)	(−4.933)	(−4.191)	(5.166)	(4.280)	(4.212)	(2.939)	(2.679)
SIZE	−0.543	−0.057	−0.578	−0.247	−0.446***	−0.315***	−0.147***	−0.010	0.075
	(−1.025)	(−1.037)	(−−1.078)	(−0.853)	(−9.260)	(−5.711)	(−3.213)	(−0.209)	(1.551)
R(12-2)					0.497***	0.359***	0.315***	0.298***	0.262***
					(7.780)	(6.277)	(5.528)	(5.201)	(4.287)
R(1-1)					−0.711***	−0.800***	−0.724***	−0.671***	−0.648***
					(−13.087)	(−13.759)	(−13.232)	(−12.729)	(−12.487)
R(60-13)					0.170***	0.112**	0.057	−0.004	−0.032
					(3.640)	(2.375)	(1.229)	(−0.088)	(−0.626)
Illiquidity					−0.352***	−0.476***	−0.240***	0.004	0.101**
					(−6.197)	(−7.993)	(−5.253)	(0.081)	(2.170)
IdioVol					1.394***	0.774***	0.320***	−0.146	−0.394***
					(7.937)	(5.747)	(2.804)	(−1.403)	(−3.774)
Intercept	−0.516***	−0.261**	−0.229*	−0.213***	−0.047	−0.024	0.004	0.034	0.055
	(−3.384)	(−2.058)	(−1.872)	(−2.657)	(−0.189)	(−0.100)	(0.018)	(0.138)	(0.224)

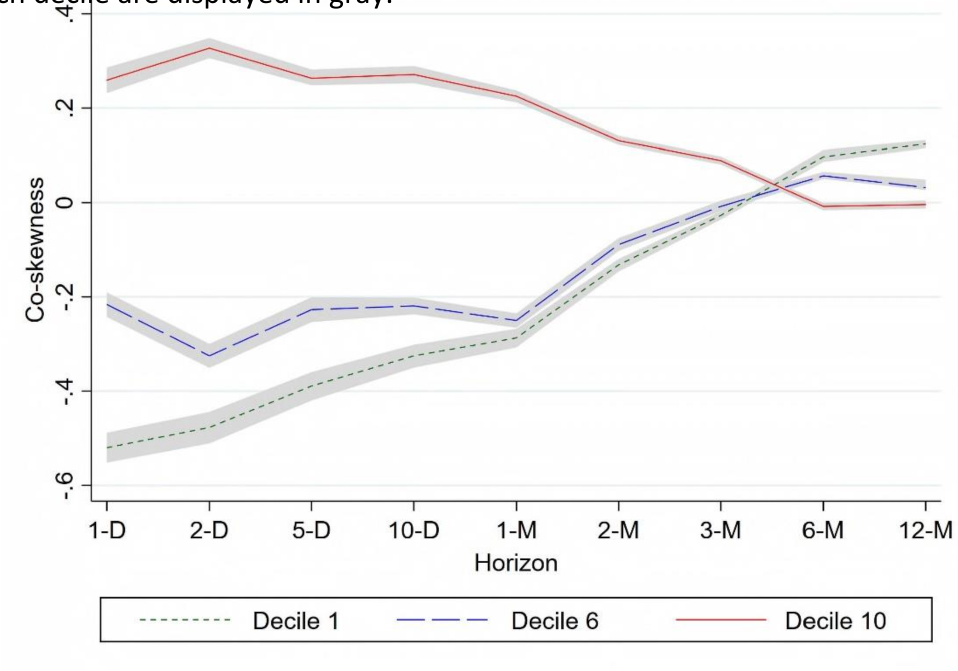
(continued)

	Panel B: Sub-sample, CRSP 1962–1989					Panel C: Sub-sample, CRSP 1990–2020				
	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
	1M	2M	3M	6M	12M	1M	2M	3M	6M	12M
MKT	−0.574***	−0.428***	−0.301***	−0.134*	−0.079	−0.179**	−0.170**	−0.114	−0.022	0.000
	(−7.790)	(−5.522)	(−4.030)	(−1.887)	(−1.195)	(−2.572)	(−2.247)	(−1.577)	(−0.418)	(0.007)
CSK	0.254***	0.134**	−0.013	0.099	0.043	0.288***	0.044	−0.083	−0.019	0.066
	(4.287)	(2.173)	(−0.188)	(1.506)	(0.67)	(4.052)	(0.600)	(−0.949)	(−0.270)	(0.874)
CKT	−0.294***	0.203**	0.180**	0.065	0.07	−0.129*	0.013	0.016	−0.071	−0.062
	(−7.014)	(2.523)	(2.306)	(1.014)	(1.311)	(−1.789)	(0.240)	(0.313)	(−0.887)	(−0.975)
B/M	0.283***	0.224***	0.188***	0.154***	0.149***	0.159***	0.199***	0.098**	0.056	0.052
	(5.968)	(4.950)	(4.227)	(3.312)	(3.031)	(2.670)	(2.603)	(2.236)	(1.264)	(1.129)
SIZE	−0.564***	−0.305***	−0.156**	−0.027	−0.049	−0.364***	−0.322***	−0.141**	−0.002	−0.094
	(−7.626)	(−4.088)	(−2.093)	(−0.356)	(−0.624)	(−5.905)	(−4.128)	(−2.428)	(−0.035)	(−1.510)
R(12-2)	0.678***	0.467***	0.402***	0.402***	0.343***	0.371***	0.285***	0.254***	0.226***	0.205**
	(7.972)	(6.398)	(5.564)	(5.695)	(4.548)	(4.254)	(3.496)	(3.118)	(2.743)	(2.320)
R(1-1)	−0.912***	−1.084***	−0.979***	−0.909***	−0.889***	−0.572***	−0.603***	−0.548***	−0.506***	−0.481***
	(−10.323)	(−11.675)	(−11.139)	(−10.692)	(−10.682)	(−9.089)	(−9.533)	(−9.078)	(−8.659)	(−8.357)
R(60-13)	0.208***	0.103*	0.018	−0.022	−0.012	0.143**	0.118*	0.084	0.009	−0.046
	(2.925)	(1.809)	(0.321)	(−0.390)	(−0.199)	(2.324)	(1.701)	(1.239)	(0.126)	(−0.604)
Illiquidity	−0.625***	−0.723***	−0.427***	−0.073	0.074	−0.162***	−0.305***	−0.111**	0.057	0.119*
	(−7.084)	(−8.433)	(−6.325)	(−1.130)	−1.222	(−2.604)	(−4.120)	(−1.986)	(0.994)	(1.804)
IdioVol	2.031***	1.072***	0.511***	−0.076	−0.355***	0.954***	0.568***	0.188	−0.195	−0.421***
	(8.312)	(5.886)	(3.348)	(−0.573)	(−2.668)	(4.188)	(3.067)	(1.179)	(−1.296)	(−2.792)
Intercept	−0.239	−0.176	−0.137	−0.09	−0.049	0.086	0.081	0.103	0.119	0.126
	(−0.585)	(−0.435)	(−0.338)	(−0.221)	(−0.120)	(0.275)	(0.268)	(0.339)	(0.395)	(0.420)

Note: This table reports the results of the second stage of the Fama-MacBeth regressions at different investment horizons. In order to compare the results across horizons, we follow the approach of Kamara et al. (2016). In the first stage of the Fama-Macbeth regression, the moving window (15-year) k-month (or -day) exposures of MKT, CSK and CKT (βkMKT, βkCSK, and βkCKT) factors in the month t are estimated using overlapping k-month (-day) excess returns and overlapping k-month (-day) factors using the past 15-year window. For all betas in the regression of month t, the average beta of the firms in the beta decile is used for the firm beta. The variable B/M is the book-to-market ratio of month t. We use the book value of the fiscal year ending in year y-1 and market value in December of year y-1 for the 12 months from July of year y to June of year y+1. The variable SIZE is the natural logarithm of market capitalization measured at the end of month t-1. For longer horizons based on monthly returns, more co-variates such as the momentum (R12, 2) factor (i.e. 11-month cumulative return in months [t-12, t-2]), the short-term price reversal (R1, 1) factor, the long-term price reversal (R60, 13) factor, horizon-adjusted illiquidity ratio and idiosyncratic volatility are controlled for. All independent variables are standardized to a mean of 0 and a standard deviation of 1 in each month. ***, ** and * stand for 99%, 95% and 90% significance level.

Figure 1:

Co-skewness of Representative Size Deciles (Decile 1, 6 and 10) across Horizons. Stocks during 1962 to 2020 are sorted into 10 size deciles. Co-skewness estimates of representative size deciles (Decile 1, 6 and 10) across horizons are illustrated. The first, Decile 1, contains the smallest firms and the last, Decile 10, contains the largest. For each individual stock of each decile across horizons, we estimate co-skewness each month using the previous 15-year periods. This figure details the average across all estimated decile stock co-skewness estimates. The distribution of other estimates for each decile are displayed in gray.



APPENDIX A:

Proof of Proposition 1: The Horizon Effect on Co-skewness (Equation 3)

Proof. Given that $\varepsilon iT_{,t}$ in Equation 3 is the residual previously extracted from the CAPM re- gression and $\varepsilon mT,t$ is the deviation of the excess market return in month t from the average value over the corresponding window, it follows:

$$\begin{cases} \bar{\epsilon}_{\mathrm{iT,t}} = 0 using OLS regression \\ \bar{\epsilon}_{\mathrm{mT,t}} = \bar{R}_{\mathrm{mT,t}} - \bar{R}_{\mathrm{mT,t}} = 0 \end{cases}$$

Thus, co-skewness as in Equation 2 can be re-written as:

$$\gamma_{\mathrm{iT,t}} = \frac{E[\epsilon_{\mathrm{iT,t}}\epsilon^2_{\mathrm{mT,t}}]}{\sqrt{E[\epsilon^2_{\mathrm{iT,t}}]}E[\epsilon^2_{\mathrm{mT,t}}]} = \frac{E[(\epsilon_{\mathrm{iT,t}}-\bar{\epsilon}_{\mathrm{iT,t}})(\epsilon^2_{\mathrm{mT,t}}-\bar{\epsilon}^2_{\mathrm{mT,t}})]}{\sqrt{E[(\epsilon^2_{\mathrm{iT,t}}-\bar{\epsilon}^2_{\mathrm{iT,t}})]}E[(\epsilon^2_{\mathrm{mT,t}}-\bar{\epsilon}^2_{\mathrm{mT,t}})]}. \tag{A.1}$$

Given that $E\ [(\varepsilon iT,t\ -\ \bar{\varepsilon}\ iT,t)]\ =\ \bar{\varepsilon}\ iT,t\ -\ \bar{\varepsilon}\ iT,t\ =\ 0$, the numerator can be derived as

$$\begin{aligned} E[(\epsilon_{\mathrm{iT,t}} - \bar{\epsilon}_{\mathrm{iT,t}})(\epsilon^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}})] &= E[(\epsilon_{\mathrm{iT,t}} - \bar{\epsilon}_{\mathrm{iT,t}})(\epsilon^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}} + \bar{\epsilon}^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}})] \\ &= E[(\epsilon_{\mathrm{iT,t}} - \bar{\epsilon}_{\mathrm{iT,t}})(\epsilon^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}})] + E[(\epsilon_{\mathrm{iT,t}} - \bar{\epsilon}_{\mathrm{iT,t}})(\bar{\epsilon}^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}})] \\ &= E[(\epsilon_{\mathrm{iT,t}} - \bar{\epsilon}_{\mathrm{iT,t}})(\epsilon^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}})] + E[\epsilon_{\mathrm{iT,t}} - \bar{\epsilon}_{\mathrm{iT,t}}] \cdot [\bar{\epsilon}^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}}] \\ &= E[(\epsilon_{\mathrm{iT,t}} - \bar{\epsilon}_{\mathrm{iT,t}})(\epsilon^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}})] + 0 \cdot [\bar{\epsilon}^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}}] \\ &= cov(\epsilon_{\mathrm{iT,t}}, \epsilon^2_{\mathrm{mT,t}}). \end{aligned} \tag{A.2}$$

Similarly, $\sqrt{E[(\epsilon^2_{\mathrm{iT,t}} - \bar{\epsilon}^2_{\mathrm{iT,t}})]} = \sigma(\epsilon_{\mathrm{iT,t}})$ and $E[(\epsilon^2_{\mathrm{mT,t}} - \bar{\epsilon}^2_{\mathrm{mT,t}})] = var(\epsilon_{\mathrm{mT,t}})$

Therefore, with $\varepsilon iT,t\ =\ RiT,t\ -\ \beta iT,tRmT,t\ -\ \alpha\hat{}\ iT,t$ and $\varepsilon mT,t\ =\ RmT,t - \bar{R}mT,t,$ co-skewness can be further refined as follows:

$$\gamma_{iT,t} = \frac{\mathrm{cov}\,(\varepsilon_{iT,t},\varepsilon^2_{mT,t})}{\sigma(\varepsilon_{iT,t})\cdot\mathrm{var}\,(\varepsilon_{mT,t})} = \frac{\mathrm{cov}\,(R_{iT,t}-\hat{\beta}_{iT,t}R_{mT,t}-\hat{\alpha}_{iT,t},R^2_{mT,t}-2\bar{R}_{mT,t}R_{mT,t}+\bar{R}^2_{mT,t})}{\sqrt{\mathrm{var}\,(R_{iT,t}-\hat{\beta}_{iT,t}R_{mT,t}-\hat{\alpha}_{iT,t})}\cdot\mathrm{var}\,(R_{mT,t}-\bar{R}_{mT,t})}. \tag{A.3}$$

We refer to returns of the shortest horizon (1-Day returns in this study) as unit returns. Using logarithmic returns, it follows that $R_{iT,t} = \sum_{j=0}^{T-1} R_{il,(t-j)}$, i.e., any return $R_{iT,t}$ can be expressed as the sum of unit returns. This also applies to market returns, $R_{mT,t}$. That is, $R_{mT,t} = \sum_{j=0}^{T-1} R_{ml,(t-j)}$. Accordingly, $R_{mT,t}^2 = \left(\sum_{j=0}^{T-1} R_{ml,(t-j)}\right)^2 = \sum_{k=0}^{T-1}\sum_{l=0}^{T-1} R_{ml,(t-k)} \cdot R_{ml,(t-l)}$. Given that the covariance between a constant and a variable is zero and

$$\begin{aligned} & \operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right) - \hat{\beta}_{iT,t} \operatorname{cov}\left(R_{mT,t}, R_{mT,t}\right) \\ = \quad & \operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right) - \frac{\operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right)}{\operatorname{cov}\left(R_{mT,t}, R_{mT,t}\right)} \cdot \operatorname{cov}\left(R_{mT,t}, R_{mT,t}\right) = 0, \end{aligned}$$

we can now rewrite the numerator of Equation A.3 as:

$$\begin{aligned} & \operatorname{cov}\left(R_{iT,t} - \hat{\beta}_{iT,t} R_{mT,t} - \hat{\alpha}_{iT,t}, R_{mT,t}^2 - 2\bar{R}_{mT,t} R_{mT,t} + \bar{R}_{mT,t}^2\right) \\ & = \operatorname{cov}\left(R_{iT,t} - \hat{\beta}_{iT,t} R_{mT,t}, R_{mT,t}^2 - 2\bar{R}_{mT,t} R_{mT,t}\right) \\ & = \operatorname{cov}\left(R_{iT,t}, R_{mT,t}^2\right) - \hat{\beta}_{iT,t} \operatorname{cov}\left(R_{mT,t}, R_{mT,t}^2\right) \\ & \quad - 2 \cdot \bar{R}_{mT,t}\left[\operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right) - \hat{\beta}_{iT,t} \operatorname{cov}\left(R_{mT,t}, R_{mT,t}\right)\right] \\ & = \operatorname{cov}\left(R_{iT,t}, R_{mT,t}^2\right) - \hat{\beta}_{iT,t} \operatorname{cov}\left(R_{mT,t}, R_{mT,t}^2\right) \\ & = \sum_{j=0}^{T-1T-1} \sum_{k=0}^{T-1} \sum_{l=0}^{T-1} \operatorname{cov}\left(R_{il,t-j}, R_{ml,(t-k)} \cdot R_{ml,(t-l)}\right) \\ & \quad - \hat{\beta}_{iT,t} \sum_{j=0}^{T-1} \sum_{k=0}^{T-1} \sum_{l=0}^{T-1} \operatorname{cov}\left(R_{ml,t-j}, R_{ml,(t-k)} \cdot R_{ml,(t-l)}\right). \end{aligned}$$

(A.4)

where $\hat{\beta}_{iT,t}$ can also be written as a function of unit returns,

$$\hat{\beta}_{iT,t} = \frac{\operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right)}{\operatorname{var}\left(R_{mT,t}\right)} = \frac{\sum_{k=0}^{T-1} \sum_{l=0}^{T-1} \operatorname{cov}\left(R_{i1,(t-k)}, R_{ml,(t-u)}\right)}{\sum_{k=0}^{T-1} \sum_{l=0}^{T-1} \operatorname{cov}\left(R_{ml,(t-k)}, R_{ml,(t-u)}\right)}$$

Similarly, for the two terms in the denominator, we have:

$$\begin{aligned} & \sqrt{\operatorname{var}\left(R_{iT,t} - \hat{\beta}_{iT,t} R_{mT,t} - \hat{\alpha}_{iT,t}\right)} \\ & = \sqrt{\operatorname{cov}\left(R_{iT,t} - \hat{\beta}_{iT,t} R_{mT,t} - \hat{\alpha}_{iT,t}, R_{iT,t} - \hat{\beta}_{iT,t} R_{mT,t} - \hat{\alpha}_{iT,t}\right)} \\ & = \sqrt{\operatorname{cov}\left(R_{iT,t}, R_{iT,t}\right) - \hat{\beta}_{iT,t} \operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right) - \hat{\beta}_{iT,t}\left[\operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right) - \hat{\beta}_{iT,t} \operatorname{cov}\left(R_{mT,t}, R_{mT,t}\right)\right]} \\ & = \sqrt{\sum_{k=0}^{T-1T-1} \sum_{u=0} \left[\operatorname{cov}\left(R_{i1,(t-k)}, R_{i1,(t-u)}\right) - \hat{\beta}_{iT,t} \operatorname{cov}\left(R_{i1,(t-k)}, R_{ml,(t-u)}\right)\right]}. \end{aligned}$$

(A.5)

and

$$\operatorname{var}\left(R_{mT,t}-\bar{R}_{mT,t}\right)=\operatorname{cov}\left(R_{mT,t},R_{mT,t}\right)=\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}\operatorname{cov}\left(R_{ml,(t-k)},R_{m1,(t-u)}\right). \tag{A.6}$$

Therefore, co-skewness can be decomposed into sums of intertemporal cross-covariance formed using unit-returns, by using A.4 in the numerator, and A.5 and A.6 in the denominator. That is, co-skewness measured with longer-horizon returns is a function of the length of the horizon T, the intertemporal auto-covariance of asset unit returns and of market unit returns, the intertemporal cross-covariance between returns of the asset and the second-order of market returns, and the estimated betas at horizon T :

$$\gamma_{iT,t}=\frac{\operatorname{cov}\left(\varepsilon_{iT,t},\varepsilon_{mT,t}^{2}\right)}{\sigma\left(\varepsilon_{iT,t}\right)\cdot\operatorname{var}\left(\varepsilon_{mT,t}\right)}$$
$$=\frac{\sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1}\operatorname{cov}\left(R_{i1,t-j},R_{ml,(t-k)}\cdot R_{ml,(t-l)}\right)-\hat{\beta}_{iT,t}\sum_{j=0}^{T-1}\sum_{k=0}^{T-1}\sum_{l=0}^{T-1}\operatorname{cov}\left(R_{ml,t-j},R_{ml,(t-k)}\cdot R_{ml,(t-l)}\right)}{\sqrt{\sum_{k=0}^{T-1T-1}\sum_{u=0}^{T-1}\left[\operatorname{cov}\left(R_{il,(t-k)},R_{il,(t-u)}\right)-\hat{\beta}_{iT,t}\operatorname{cov}\left(R_{il,(t-k)},R_{ml,(t-u)}\right)\right]}\cdot\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}\operatorname{cov}\left(R_{ml,(t-k)},R_{ml,(t-u)}\right)}. \tag{A.7}$$

APPENDIX B:

Proof of Proposition 2: the "Scaling Law" (Equation 5)

Proof. There are four sums of covariance terms and a horizon-dependent beta estimate in the decomposition of co-skewness into terms relating to unit returns, according to Equation A.7:

1. the sum of auto-covariances of asset returns,

$$\sum_{k=0}^{T-1}\sum_{u=0}^{T-1}\operatorname{cov}\left(R_{i1,(t-k)},R_{i1,(t-u)}\right),$$

which includes T contemporaneous auto-covariances (corresponding to $k=u$) and $T(T-1)$ intertemporal auto-covariances (when $k\neq u$) of asset returns.

2. the sum of auto-covariances of market returns,

$$\sum_{k=0}^{T-1}\sum_{u=0}^{T-1} \operatorname{cov}\left(R_{ml,(t-k)}, R_{ml,(t-u)}\right),$$

which includes T contemporaneous auto-covariances (corresponding to $k = u$) and $T(T-1)$ intertemporal auto-covariances of market returns.

3. the sum of cross-covariances between returns of asset i and a second-order term using market returns,

$$\sum_{k=0}^{T-1}\sum_{u=1}^{T-1}\sum_{j=0}^{T-1} \operatorname{cov}\left(R_{i1,t-k}, R_{ml,(t-u)} \cdot R_{ml,(t-j)}\right),$$

which includes T contemporaneous cross-covariances (corresponding to $k = u = j$), and $T * T(T-1)$ intertemporal cross-covariances.

4. the sum of cross-covariances between market returns and a second-order term using market returns,

$$\sum_{k=0}^{T-1}\sum_{u=1}^{T-1}\sum_{j=0}^{T-1} \operatorname{cov}\left(R_{ml,t-k}, R_{ml,(t-u)} \cdot R_{ml,(t-j)}\right),$$

which includes T contemporaneous cross-covariances (corresponding to $k = u = j$), and $T * T(T-1)$ intertemporal cross-covariances.

5. the beta estimate

$$\hat{\beta}_{iT,t} = \frac{\operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right)}{\operatorname{var}\left(R_{mT,t}\right)} = \frac{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} \operatorname{cov}\left(R_{i1,(t-k)}, R_{ml,(t-u)}\right)}{\sum_{k=0}^{T-1}\sum_{l=0}^{T-1} \operatorname{cov}\left(R_{ml,(t-k)}, R_{ml,(t-u)}\right)},$$

Given stationary returns and for unconditional estimates, the four contemporaneous covariance terms equal to $\operatorname{cov}\left(R_{i1,t}, R_{il,t}\right), \operatorname{cov}\left(R_{ml,t}, R_{ml,t}\right), \operatorname{cov}\left(R_{i1,t}, R_{ml,t}^2\right)$ and $\operatorname{cov}\left(R_{ml,t}, R_{ml,t}^2\right)$, respectively and there are T of each one of the these.

In the "perfect market" proposed by Hawawini (1980b), securities display no intertemporal cross- and auto-correlations. In this case, beta is constant over all horizons, as documented by Hawawini (1980b),

$$\beta_{iT,t} = \frac{\operatorname{cov}\left(R_{iT,t}, R_{mT,t}\right)}{\operatorname{var}\left(R_{mT,t}\right)} = \beta_{i1,t} \cdot \frac{T + \sum_{s=1}^{T-1} (T-s)\frac{\rho_{im,t}^{+s} + \rho_{im,t}^{-s}}{\rho_{im,t}}}{T + 2\sum_{s=1}^{T-1} (T-s)\rho_{mm,t}^{s}} = \beta_{i1,t}. \tag{A.8}$$

Similarly, in the absence of the intertemporal cross- and auto-covariance terms described above, Equation A.7 can be rewritten as:

$$
\begin{aligned}
\gamma_{iT,t} &= \frac{\operatorname{cov}\left(\varepsilon_{iT,t}, \varepsilon_{mT,t}^{2}\right)}{\sigma\left(\varepsilon_{iT,t}\right) \cdot \operatorname{var}\left(\varepsilon_{mT,t}\right)} \\
&= \frac{T \cdot \operatorname{cov}\left(R_{i1,t}, R_{m1,t} \cdot R_{m1,t}\right) - \hat{\beta}_{i1,t} T \cdot \operatorname{cov}\left(R_{m1,t}, R_{m1,t} \cdot R_{m1,t}\right)}{\sqrt{T \cdot \operatorname{cov}\left(R_{i1,t}, R_{i1,t}\right) - \hat{\beta}_{i1,t} \operatorname{cov}\left(R_{i1,t}, R_{m1,t}\right)} \cdot T \cdot \operatorname{cov}\left(R_{m1,t}, R_{m1,t}\right)} \\
&= \frac{1}{\sqrt{T}} \cdot \gamma_{i1,t}.
\end{aligned}
\qquad \text{(A.9)}
$$

That is, in contrast to the earlier findings of the horizon effect on betas in the literature, in a market where intertemporal cross- and auto-covariance terms are equal to zero, coskewness is still horizon dependent. In this extreme case, with the lengthening of the measurement horizon, unconditional coskewness converges to zero, highlighting a "scaling law" of co-skewness.

APPENDIX C:

Robustness Check on Co-kurtosis

In this paper, we focus on testing the sensitivity of stock co-skewness to the investment horizon. In light of the well-documented horizon dependence of the 2^{th} order comoment and our findings relating to the 3^{rd} order comoment, it is reasonable to expect that higher-order comoments may also be horizon sensitive. Specifically, Fang and Lai (1997) and Dittmar (2002) suggest a four-moment capital asset pricing model by including co-kurtosis. Co-kurtosis is defined as the component of an asset's kurtosis that is related to the market portfolio's kurtosis.

Similar to Kostakis et al. (2012), we estimate co-kurtosis as an extension to the co-skewness measure of Harvey and Siddique (2000):

$$\delta_{iT,t} = \frac{E[\varepsilon_{iT,t}\varepsilon_{mT,t}^3]}{\sqrt{E[\varepsilon_{iT,t}^2]}E[\varepsilon_{mT,t}^3]} \tag{A.10}$$

where $\varepsilon_{iT,t} = [R_{iT,t} - R_{fT,t}] - [\alpha_{iT,t} + \beta_{iT,t}(R_{mT,t} - R_{fT,t})]$ is the residual previously extracted in Equation 1 from a CAPM regression and $\varepsilon_{mT,t}$ is the deviation of the excess market return in month t from the average over the corresponding window.

Empirical results are shown in Table A.1. Panel A shows the estimated co-kurtosis for size deciles, which indicates the existence of the horizon effect on co-kurtosis. As shown in Panel B, the magnitude of the horizon effect on co-kurtosis, when measured using standard deviations and estimation bias, are also significant. While no obvious pattern is evident at short horizons, at longer horizons we provide evidence of decreasing co-skewness from the smallest to second largest portfolio. These findings shed further doubt on the empirical estimation of higher-order moments.

TABLE A.1 ABOUT HERE

These findings highlight the importance and potential implications of the horizon effect on asset pricing. Our detailed findings relating to the horizon effect on co-skewness is not only a replication or sequel to those on alpha and beta, but also a pervasive concern with implications throughout many aspects of asset pricing.

APPENDIX D:

Kraus and Litzenberger (1976) Co-skewness Estimates and Risk Premium

Throughout this paper, we focus on the horizon effect on co-skewness estimated using the approach of Harvey and Siddique (2000). Earlier research, including Rubinstein (1973) and Kraus and Litzenberger (1976) also extend the static CAPM to nonlinear forms of the risk-return trade-off by considering systematic skewness. To ensure our findings are robust to the estimation method, we also examine whether higher-order comoments estimated using the approach of Kraus and Litzenberger (1976) are also sensitive to the measurement horizon. Accordingly, Kraus and Litzenberger (1976) co-skewness, $\gamma_{iM,T}^{KL}$, is a measure of systematic skewness of the risky asset i with respect to the market portfolio M and is given by

$$\gamma_{iM,T}^{KL} = C_{i3}\left(R_{i,T}, R_{M,T}\right) = \frac{E\left[\left(R_{i,T}-\bar{R}_{i,T}\right)\left(R_{M,T}-\bar{R}_{M,T}\right)^2\right]}{E\left[\left(R_{M,T}-\bar{R}_{M,T}\right)^3\right]} \quad \text{(A.11)}$$

Using the same sample, we test the horizon effect on Kraus and Litzenberger (1976) coskewness across 10 size deciles, as well as the three measures of estimation variation. Results are shown in Table A.2, indicating a consistent horizon effect. The magnitudes of the horizon effect on Kraus and Litzenberger (1976) co-skewness are also inversely related to the market capitalization, for all securities with greater expected trading delays than the weighted average trading delay in the market. Moreover, Kraus and Litzenberger (1976) co-skewness is a measure closer to the traditional definition of beta. Therefore, these estimates converge to 1 when returns of longer horizons are used.

TABLE A.2 ABOUT HERE

Furthermore, Andries et al. (2019) find "dynamically inconsistent preferences in which risk aversion decreases with the temporal horizon". Thus, it is reasonable to expect that decreasing risk aversion with longer horizon will impact on the risk premium for co-skewness. The Harvey and Siddique (2000) measure of co-skewness permits us to accommodate nonincreasing absolute risk aversion but not global risk aversion. Existing studies such as Post and Levy (2005) and Post et al. (2008) have shown significant evidence that co-skewness has minimal explanatory power after imposing risk aversion. In particular, Post et al. (2008) documented consistent findings across horizons. Therefore, although risk aversion and its impact on the estimation of co-skewness across horizons is not our main focus, we replicate the test using the Kraus and Litzenberger (1976) co-skewness estimate, with results shown in Table A.3.

TABLE A.3 ABOUT HERE

Following Post et al. (2008), we estimate the Euler equation for two different models. Given a cubic utility function, $u(x \mid \theta) = 1 + \theta_1 x^2 + \theta_2 x^3$, the first model imposes no estimation restrictions requiring the value-weighted average of the pricing errors to equal zero:

$$\mathbf{m}(\theta)^T \tau = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t^T \tau + 2\theta_1 \frac{1}{T}\sum_{t=1}^{T} (\mathbf{x}_t^T \tau)^2 + 3\theta_2 \frac{1}{T}\sum_{t=1}^{T} (\mathbf{x}_t^T \tau)^3 = 0 \qquad \text{(A.12)}$$

The second model adds the condition of risk aversion over the sample range, i.e. $2\theta_1 + 6\theta_2 b_+ \leq 0$ where $b+$ is the upper bound of the returns during the sample period. For each horizon, Table A.3 reports the estimated utility parameters (θ_1 and θ_2) for both models and the associated risk premiums (ρ_1 and ρ_2) are computed as:

$$\begin{aligned}\rho_1 &\equiv \frac{-E[u''(\mathbf{x}^T\tau|\theta)]E\left[\left(\mathbf{x}^T\tau-\mu^T\tau\right)^2\right]}{E[u'(\mathbf{x}^T\tau|\theta)]} = \frac{-(2\theta_1+6\theta_2\mu^T\tau)E\left[\left(\mathbf{x}^T\tau-\mu^T\tau\right)^2\right]}{1+2\theta_1\mu^T\tau+3\theta_2 E[(\mathbf{x}^T)^2]},\\ \rho_2 &\equiv \frac{-1/2E[u'''(\mathbf{x}^T\tau|\theta)]E\left[\left(\mathbf{x}^T\tau-\mu^T\tau\right)^2\right]}{E[u'(\mathbf{x}^T\tau|\theta)]} = \frac{-3\theta_2\mu^T\tau)E\left[\left(\mathbf{x}^T\tau-\mu^T\tau\right)^2\right]}{1+2\theta_1\mu^T\tau+3\theta_2 E[(\mathbf{x}^T)^2]}.\end{aligned} \qquad \text{(A.13)}$$

Furthermore, the table reports the J-statistics which provide an indication as to whether the Euler equation holds. We analyze the efficiency of the market portfolio relative to our 10 size deciles over our sample period July 1963 - December 2020. We compare the model that imposes the condition of risk aversion over the sample range, with the model without this restriction. Similar to the literature, using shorter-horizon returns results in significant evidence that when restricting the risk-aversion, the co-skewness premium approaches zero. However, we also find that when longer horizons (e.g. longer than 4-month) are considered, co-skewness has a smaller risk premium even if risk-aversion is not restricted. Adding the restriction of risk-aversion, while still approaching zero, has less impact on the co-skewness premium at long-horizons.[11] This helps to reconcile our findings with the previous literature indicating that estimation of the co-skewness premium is highly dependent upon the empirical set-up.

[11] The null hypothesis of J-statistic is significantly rejected when 12M returns are used.

Table A.1 Co-kurtosis of size deciles across horizons

		Panel A: Mean co-kurtosis of each portfolio									Panel B: Measures of estimation variations		
	Mkt. Cap (Million)	1D	2D	5D	10D	1M	2M	3M	6M	12M	Standard deviation	Repeated measures	Estimation bias ANOVA
Decile 1	42.710	−3.509	−2.057	−1.287	−0.358	−0.693	−0.392	−0.089	0.417	0.202	1.249***	0.792	8.020***
Decile 2	113.237	−1.620	−1.232	−0.119	−0.041	−0.679	−0.280	0.011	0.264	0.153	0.651***	0.828	5.555***
Decile 3	200.913	−0.285	−0.565	−0.186	0.062	−0.638	−0.225	0.030	0.185	0.091	0.291***	0.835	4.209***
Decile 4	318.976	−1.131	−0.563	−0.108	−0.065	−0.574	−0.135	0.141	0.251	0.121	0.445***	0.880	4.219***
Decile 5	485.273	−0.457	−0.727	−0.116	−0.145	−0.649	−0.281	−0.028	−0.103	0.062	0.279***	0.883	3.550***
Decile 6	723.657	−1.368	−0.376	0.909	0.252	−0.599	−0.178	0.125	−0.134	0.044	0.626***	0.877	5.605***
Decile 7	1114.856	−0.565	−1.967	−1.168	−0.545	−0.789	−0.239	−0.018	−0.319	−0.129	0.611***	0.825	4.475***
Decile 8	1895.457	−2.933	−0.764	0.236	−0.521	−0.803	−0.288	−0.064	−0.352	−0.152	0.926***	0.887	5.842***
Decile 9	3842.743	−2.938	−3.096	−0.959	−0.646	−0.205	−0.178	−0.287	−0.542	−0.061	1.182***	0.835	7.409***
Decile 10	39384.100	1.058	0.853	0.804	0.492	0.688	0.283	0.121	0.041	0.045	0.383***	0.802	3.110***

Note: Co-kurtosis estimated using value-weighted portfolio returns and the market index are illustrated in Panel A. Stocks are allocated into 10 size deciles based on NYSE breakpoints. Daily returns of stocks from August, 1962 to December, 2020 on the CRSP daily tape are used when estimating 1-day, 2-day, 5-day and 10-day co-kurtosis, and similarly monthly returns of stocks from the CRSP monthly tape are used to estimate 1-month and longer-horizon co-kurtosis. The first, "Decile 1", contains the smallest firms and the last, "Decile 10", contains the largest. For each size decile and each horizon, we estimate co-kurtosis at each month from August 1977 to December 2020 using the previous 15-year period. This table details the average across all portfolio co-kurtosis estimates. In Panel B, the standard deviation, repeated measure ANOVA tests and estimation bias estimates, as in Equation 6, are used in detecting the magnitude of the horizon effect. ***, ** and * indicate that the null hypothesis that the estimates are equal to zero is rejected at the 99%, 95% or 90% level, respectively.

Table A.2 Co-skewness of size deciles across horizons. Kraus and Litzenberger (1976)

		Panel A: Mean co-kurtosis of each portfolio									Panel B: Measures of estimation variations		
	Mkt. Cap (Million)	1D	2D	5D	10D	1M	2M	3M	6M	12M	Standard deviation	Repeated measures	Estimation bias ANOVA
Decile 1	42.710	0.107	1.453	1.269	1.327	1.338	1.197	1.158	1.146	1.035	0.398***	1.830**	2.240***
Decile 2	113.237	0.203	1.506	1.304	1.404	1.355	1.223	1.170	1.201	1.098	0.382***	1.847**	2.133***
Decile 3	200.913	0.226	1.471	1.312	1.386	1.377	1.230	1.180	1.212	1.081	0.371***	1.083	2.121***
Decile 4	318.976	0.178	1.384	1.258	1.308	1.327	1.207	1.167	1.201	1.075	0.366***	1.504*	1.951***
Decile 5	485.273	0.208	1.395	1.247	1.310	1.329	1.228	1.206	1.279	1.137	0.360***	1.040	1.752***
Decile 6	723.657	0.211	1.326	1.186	1.227	1.250	1.163	1.129	1.165	1.071	0.334***	1.065	1.671***
Decile 7	1114.856	0.468	1.356	1.197	1.235	1.189	1.158	1.138	1.186	1.110	0.253***	0.872	1.104***
Decile 8	1895.457	0.528	1.222	1.111	1.122	1.086	1.101	1.090	1.081	1.033	0.199***	0.855	0.833***
Decile 9	3842.743	0.845	1.094	0.987	1.067	0.990	1.028	1.038	1.044	0.970	0.073***	0.692	0.488***
Decile 10	39384.100	1.082	0.900	0.859	0.889	0.900	0.922	0.927	0.933	0.979	0.065***	1.452	0.396***

Note: Kraus and Litzenberger (1976) co-skewness estimates using value-weighted portfolios returns and market index are illustrated in Panel A. Stocks are allocated into 10 size deciles based on the NYSE breakpoints. Daily returns of stocks from August, 1962 to December, 2020 on the CRSP daily tape are used when estimating 1-day, 2-day, 5-day and 10-day co-skewness, and similarly monthly returns of stocks on the CRSP monthly tape are used to estimate 1-month and longer-horizon co-skewness. The first, Decile 1, contains the smallest firms and the last, Decile 10, contains the largest. For each size decile and each horizon, we compute the co-skewness estimate at each month from August 1977 to December 2020 using the previous 15-year periods. This table details the average across all portfolio co-skewness estimates. In Panel B, the standard deviation, repeated measure ANOVA tests and estimation bias estimates, as in Equation 6 are used in detecting the magnitude of the horizon effect. ***, ** and * indicate that the null hypothesis that the estimates are equal to zero is rejected at the 99%, 95% or 90% level, respectively.

Table A.3 Estimation results three-moment CAPM considering risk aversion.

Horizon	Risk aversion	θ_1	θ_2	ρ_1	ρ_2	J
1D	Yes	−0.0135 (0.0000)	0.0004 (0.4491)	0.0275	0.0006	0.0009 (0.1897)
	No	−0.0042 (0.1626)	0.0133 (0.0011)	0.0061	0.0220	0.0004 (0.5800)
2D	Yes	−0.0090 (0.0767)	0.0002 (0.4865)	0.0395	0.0021	0.0064 (0.3767)
	No	−0.0005 (0.4933)	0.0041 (0.0249)	0.0000	0.0416	0.0039 (0.7197)
5D	Yes	−0.0135 (0.0076)	0.0002 (0.4368)	0.1366	0.0041	0.0031 (0.3313)
	No	−0.0070 (0.2992)	0.0056 (0.0419)	0.0434	0.0974	0.0019 (0.6976)
10D	Yes	−0.0135 (0.2087)	0.0001 (0.4653)	0.2735	0.0079	0.0056 (0.4048)
	No	−0.0077 (0.3459)	0.0044 (0.0266)	0.0716	0.2098	0.0026 (0.8743)
1M	Yes	−0.0143 (0.0061)	0.0002 (0.4456)	0.5635	0.0220	0.0108 (0.4775)
	No	−0.0117 (0.0431)	0.0041 (0.1347)	0.1441	0.4414	0.0082 (0.6752)
2M	Yes	−0.0142 (0.0107)	0.0001 (0.4782)	1.1283	0.0415	0.0181 (0.6111)
	No	−0.0183 (0.0826)	0.0052 (0.3482)	0.0006	1.1692	0.0124 (0.8257)
3M	Yes	−0.0134 (0.4794)	0.0001 (0.4696)	1.6816	0.0768	0.0999 (0.5961)
	No	−0.0138 (0.4818)	0.0026 (0.3114)	0.0153	1.7431	0.0262 (0.9471)
6M	Yes	−0.0144 (0.3579)	0.0001 (0.3889)	3.3182	0.1051	0.0458 (0.7243)
	No	−0.0192 (0.4895)	0.0006 (0.4738)	2.7981	0.6252	0.0416 (0.7761)
12M	Yes	−0.0074 (0.6680)	0.0000 (0.5715)	4.2597	0.0007	0.8327 (0.0000)
	No	−0.0074 (0.6679)	0.0000 (0.5715)	4.2604	0.0000	0.8327 (0.0000)

Note: We compare the model that imposes the condition of risk aversion over the sample range (No), with the model without this restriction (Yes) as suggested by Post et al. (2008). We estimate the parameters θ1 and θ1 of the cubic utility function $u(x|\theta) = 1 + \theta 1x^2 + \theta 2x^3$ using GMM method and calculate associated risk premiums (ρ1 and ρ2) as defined by Equation A.13. Our 10 size deciles over the sample period July 1963 - December 2020 are investigated. The table shows the GMM estimates for the parameters (p-values within brackets), the associated estimates for the risk premiums ρ1 and ρ2, and the J-statistics (p-values within brackets). J-statistic is used to test whether the null hypothesis (the pricing errors are equal to zero) is rejected to not. The null hypothesis of J-statistic is rejected when 12M returns are used.