Bernstein - von Mises Theorems for general functionals

In this work we study conditions on the prior and on the model to obtain a Bernstein von Mises Theorem for finite dimensional functionals of a curve. A Bernstein - von Mises theorem for a parameter of interest ψ essentially means that the posterior distribution of ψ asymptotically behaves like a Gaussian distribution with centering point some statistics $\hat{\psi}$ and variance V_n, where the frequentist distribution of $\hat{\psi}$ at the true distribution associated with parameter ψ is also a Gaussian with mean ψ and variance V_n. Such results are well known in parametric regular models and have many interesting implications. One such implication is the fact that it links strongly Bayesian and frequentist approaches. In particular Bayesian credible regions such as HPD regions are also asymptotically valid frequentist confidence regions, when the Bernstein von Mises Theorem is valid.

In this work we are interested in a semi-parametric setup, where the unknown parameter η is infinite dimensional and one is interested in a finite dimensional functional of it : $\psi = \psi(\eta) \in \mathbb{R}^d$. We will give a general theorem which gives some conditions ensuring the validity the of Bernstein von Mises Theorem and then some specific setups will be considered such as non linear functionals like the $\int f^2$ where f is the unknown curve or approximately linear functionals of either the density or the regression function in a non linear autoregressiv model.