Let $K$ be a field and let $M_n(K)$ denote the space of $n \times n$ matrices over $K$. Define the symmetric bilinear $f$ form on $M_n(K) \times M_n(K)$ by $f(A, B) = Tr(AB)$, where Tr denotes the trace of a matrix. Given a subspace $\mathcal{M}$ of $M_n(K)$ let $\mathcal{M}^\perp$ denote the perpendicular subspace of $\mathcal{M}$.

We show that, for a suitable choice of subspace $\mathcal{M}$, $\mathcal{M}^\perp$ sometimes has the property that it contains no elements of rank one or no elements of rank one or two. In particular, if $K$ is a field such that $M_n(K)$ admits a subspace $\mathcal{M}$ of dimension $n$ in which each non-zero element of $\mathcal{M}$ is invertible, $\mathcal{M}^\perp$ contains no elements of rank one, and its dimension $n^2 - n$ is maximal.

We consider analogues of these constructions when we restrict the form f to the subspace of symmetric matrices and consider perpendicular spaces in this smaller space. Of special interest is the case of an $n$-dimensional subspace $\mathcal{M}$ of symmetric matrices over a finite field $K$ of odd characteristic in which each non-zero element is invertible. We discuss the rank of elements in $\mathcal{M}^\perp$ for such an $\mathcal{M}$, distinguishing between when $n$ is odd and $n$ is even.