

EXPERIMENT 20

The Hall Effect

Note: Please read the Radiation Safety Regulations at the back of this book

Objectives of the Experiment

The behaviour of the Hall voltage in a sample of doped germanium is studied in three different set-ups as follows:

1. Constant control current and temperature and varying magnetic field.
2. Constant magnetic field and temperature and varying control current.
3. Constant magnetic field and control current and varying temperature.

From these measurements the sign of the charge carriers, the Hall constant, the carrier mobility and the carrier concentration can all be determined.

Theory

The Drude model of electron conduction

The relationship between the current flowing in a conducting medium and the applied voltage is given by Ohm's law which states

$$V = IR$$

This very simple law applies to a very wide range of materials such as metals, semiconductors, electrolytic solutions, weakly ionized gases etc. In metals it also applies over a very wide range of current densities from 100 amp/m². At 10¹³ amp/m² the deviation from Ohm's law is only 1%.

The conducting medium may be viewed as an assembly of atoms through which the charge carriers move when an electric field is applied to the medium. The charge carriers, whether they be electrons, ionized atoms or positive holes have thermal energy of

$$\frac{3}{2}kT$$

where k is Boltzmann's constant and T is the absolute temperature. If m is the mass of the charge carriers then the thermal velocity (\bar{v}) of the carriers may be obtained from

$$\frac{1}{2} m \bar{v}^2 = \frac{3}{2} kT \quad (20.1)$$

The flow of current through a metal wire may be pictured as follows:

The atoms are all bound into the crystal lattice. Each atom readily loses its valence electrons so that if a voltage is applied to the wire these valence electrons move along the electric field lines with a directed velocity, superimposed on their random thermal velocity. The electrons are accelerated by the electric field but periodically they collide with an atom of the crystal lattice and, as a result of the collision, lose the directed velocity acquired from the electric field. This fluctuating velocity directed along the field lines, distinct from the thermal velocity, is called the drift velocity, v_d and is given by:

$$v_d = a \tau \quad (20.2)$$

where a is the acceleration of the electrons and τ is the mean time between collisions, related to the mean free path, λ , by :

$$\tau = \frac{\lambda}{\bar{v}} \quad (20.3)$$

From Newton's 2nd law :

$$a = \frac{E \cdot e}{m} \quad (20.4)$$

where

E is the electric field strength

e is the charge on the electron

and m is the mass of the electron.

Substituting Eqs. 20.3 and 20.4 into Eq. 20.2 we get:

$$v_d = \frac{E \cdot e}{m} \frac{\lambda}{\bar{v}} \quad (20.5)$$

Electron mobility (μ) is defined as :

$$\mu = \frac{e \lambda}{m \bar{v}} \quad (20.6)$$

and

$$\mu = \frac{e}{m} \tau \text{ from Eq. 20.3}$$

The mobility is thus the drift velocity acquired by an electron per unit electric field and

$$v_d = \mu E \quad (20.7)$$

The drift of the electron in response to the applied voltage (electric field) gives rise to a current I given by :

$$I = neAv_d \quad (20.8)$$

where:

n is the number of electrons per unit volume

A is the cross-sectional area of the conductor.

Hence using Eq. 20.7:

$$I = neA\mu E \quad (20.9)$$

If l is the length of the conductor, the voltage across it is:

$$V = El \quad (20.10)$$

From Ohm's law and Eqs. 20.9 and 20.10 the resistance R is given by:

$$R = \frac{V}{I} = \frac{El}{neA\mu E}$$

$$R = \frac{l}{neA\mu} \quad (20.11)$$

R can also be written in terms of the dimensions of the conductor and its resistivity ρ

$$R = \frac{\rho l}{A} \quad (20.12)$$

Combining Eqs. 20.11 and 20.12 gives:

$$\rho = \frac{1}{ne\mu} \quad (20.13)$$

The Hall Effect

The Hall effect was first observed by an American scientist, E. H. Hall, in 1879. It is illustrated in Fig. 20.1. If a current passes through a conducting material along the X direction and there is a magnetic field in the Y direction then a voltage difference V_H appears along the Z axis, that is between the faces C & D of the material. If a charge e moves with velocity v_d through a magnetic field B then a force \vec{F} acts on the charge, where

$$\vec{F} = e\vec{v}_d \times \vec{B} \quad (20.14)$$

If the charge moves along the X-axis and the magnetic field is along the Y axis then the force is along the Z axis. If the charges are positive then the force acts in the direction of the positive Z axis. Positive charges will therefore move upwards towards face C and because an excess charge accumulates on face C with a corresponding shortage on face D, a voltage, the Hall voltage, develops in the Z direction. The Hall voltage increases until it exactly balances the force due to the magnetic field and equilibrium is reached. Hence,

$$ev_d B = \frac{eV_H}{h}$$

$$V_H = v_d B h \quad (20.15)$$

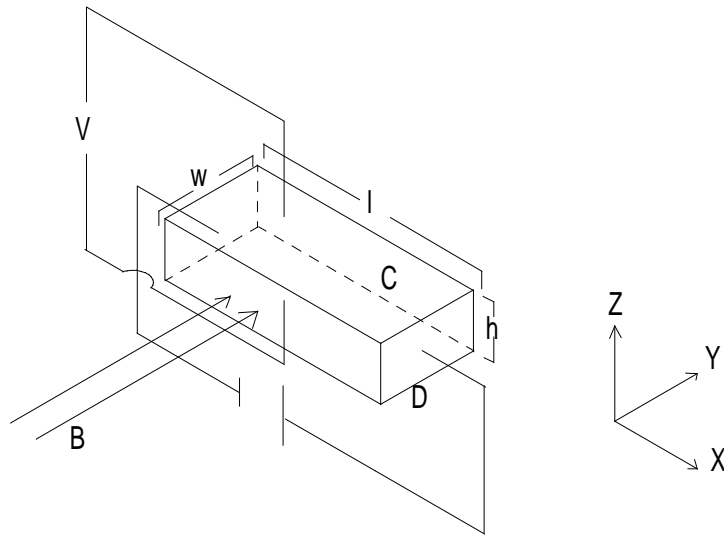


Figure 20.1: Set-up illustrating the generation of a Hall voltage in a conducting medium in the presence of a magnetic field.

where h is the thickness of the crystal in the Z-direction. This can be written in terms of the current making use of Eq. 20.8. Hence,

$$V_H = \frac{hBI}{neA} \quad (20.16)$$

From Fig. 20.1 the cross section area (A) is

$$A = wh$$

$$V_H = \frac{BI}{new} \quad (20.17)$$

The Hall constant is defined as :

$$R_H = \frac{1}{ne} \quad (20.18)$$

Substituting Eq. 20.18 in Eq.20.17

$$V_H = \frac{R_H B}{w} I \quad (20.19)$$

Eq. 20.19 can be written as follows :

$$R_H = \frac{V_H w}{IB} \quad (20.20)$$

From Eq. 20.17 it can be seen that the Hall voltage changes linearly with magnetic field and such a 'Hall probe' device is often used to measure magnetic fields. It is not practical to use metals in such devices as n is very large for these materials, making V_H very small. Of course for insulating materials, such as glass, n is very small but very large voltages are then required to increase I to

some measurable value. It was not until the manufacture of semi-conducting materials that the value of n could be set to its optimum value and the device then became a convenient method of measuring magnetic fields.

The Hall effect in semiconductors

Semiconductors have important characteristics which have led to their crucial role in the electronics revolution of the last 50 years. Although they have lower room temperature conductivity than metals, their conductivity increases as temperature increases, unlike metals, and both electrons and positive carriers ('holes') may contribute to the conductivity (see experiment on the PN junction for more details). Semiconductors can be chemically 'doped' so that there is an excess of either electrons ('n-type') or holes ('p-type') participating in conduction. Doping alters the conductivity of the semiconductor and offers many possibilities to control the behaviour of the material and is the main reason for the importance of semiconductors in modern electronic devices.

The relative contribution of electrons and holes to the conductivity changes as a function of temperature and the behaviour of the Hall voltage with temperature can be used to examine the type of charge carriers which dominate in different temperature regions.

Experimental Set-up and Procedure

A schematic diagram of the experimental set-up is shown in Fig. 20.2a.

The heater circuit is not required until Part 2 of the experiment.

The Hall crystal is a sample of either p-type or n-type germanium (depending on the apparatus being used) mounted on a plug-in board and attached to the base unit. The crystal is extremely fragile and the plug-in board should not be removed from the base unit.

The magnetic field surrounding the sample is provided by the electromagnet through which a current is passed from PSU 1. The value of the magnetic field near the sample is monitored using a Hall probe which is a precalibrated sensor that is connected via the B-box to the CASSY interface for computerised recording. The probe is very delicate and should be moved carefully into position along the rail provided so that the probe tip is mid-way along the sample.

Power supply unit 1 (PSU1) also provides the current to the sample as shown in Fig. 20.2a. The current is measured using a multimeter in series. A connection to the CASSY interface for computerised recording can be made as required.

The Hall voltage generated is measured across the sample using a multimeter in parallel. A connection to the CASSY interface for computerised recording can be made as required.

Part 1

In this part of the experiment there are 3 parameters of interest: current in the sample, magnetic field in the sample and Hall voltage generated. We wish to explore how the Hall voltage varies with each parameter separately, keeping the other constant and hence verify (or disprove!) the predictions of the classical model described in Section 20.

Firstly, set the circuit up as shown in Fig. 20.2a and carefully set the sample current using the 'current adjust' control on the base unit to a value of ~ 30 mA. Record this current value and

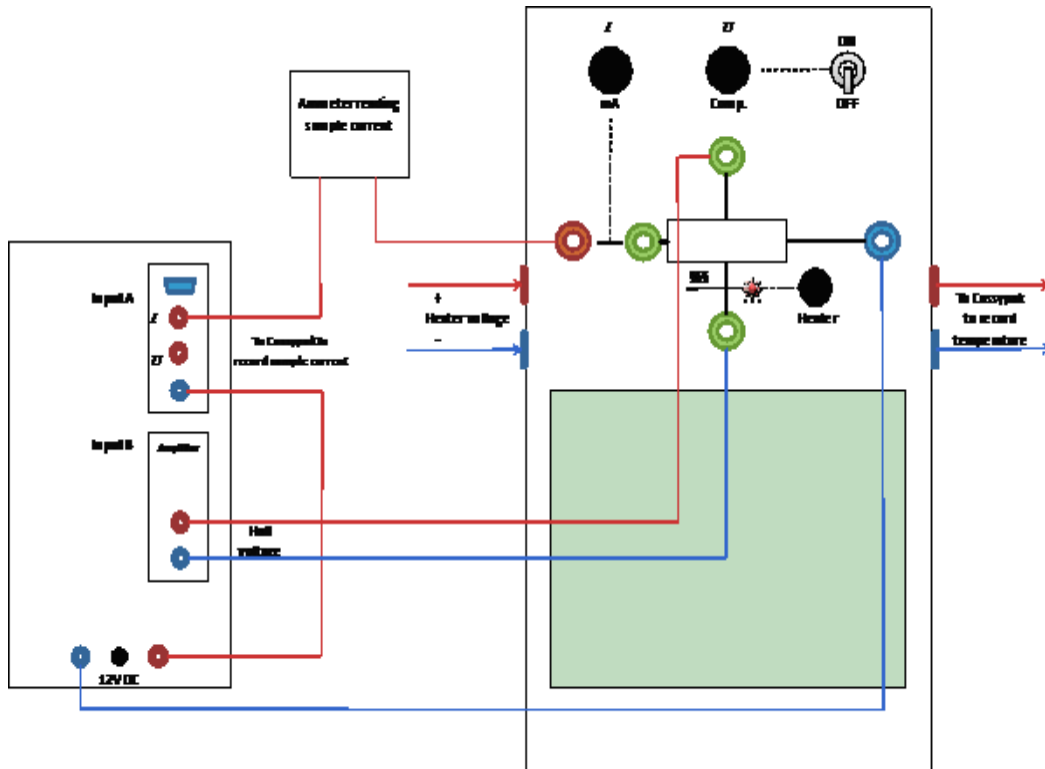


Figure 20.2a: Schematic of Hall effect experiment set-up. Not all connections shown are used in all experiments.

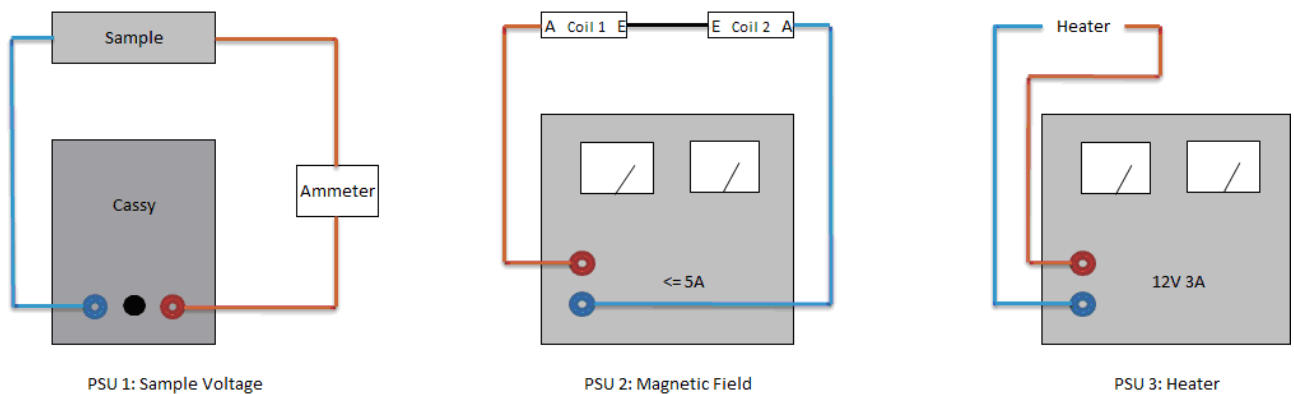


Figure 20.2b: Connections to the three power supplies. PSUs 1 and 2 are required for all experiments, PSU 3 for Part 2 only.

ensure it remains constant in this part of the experiment. Connect the Hall probe to Input A of the CASSY unit and the Hall voltage multimeter to Input B via the Amplifier box (see Panel 1 of Fig. 20.3).

Start the CASSYLab software on the PC and set up the axes for the experiment. The magnetic field should be recorded on the x-axis, since this is the controlled parameter and the measured Hall voltage should be recorded on the y-axis. Run the program. Now, increase the current through the electromagnet via PSU 2 until the maximum value is reached. Repeat the measurement a few times. How reproducible is the resulting graph? Print out your graph.

Repeat the procedure, this time keeping the magnetic field fixed at some known value. How should you determine the value of the magnetic field? Record the Hall voltage as a function of sample current (see middle panel of Fig. 20.3). Make sure the axes are correctly set for the new parameters. Again, run the experiment a few times to determine how reproducible your graph is and make a printout.

Data Analysis

From Eq. 20.19 the Hall voltage should vary linearly as a function of magnetic field or sample current. Is this prediction borne out by your data? Determine the slopes of your graphs and use Eq. 20.19 to make 2 independent estimates of the Hall coefficient, R_H , of your sample.

The width of the sample, w , is 1 mm.

What are the dominant sources of error on your results? Use the propagation of errors formula and justifiable order of magnitude estimates of the error on each measured quantity to derive an overall percentage error for each R_H value. Do not go into excessive mathematical detail. Do your two estimates of R_H agree to within the errors?

Use Eq. 20.18 to determine n , the number of charge carriers per unit volume in the sample.

Note: Undoped germanium at room temperature has $\sim 2 \times 10^{19}$ carriers per m^3 . How does your value compare with this?

Measure the resistance of the sample in the absence of the magnetic field. Using Eq. 20.12 determine ρ and hence μ (Eq. 20.13) and τ .

Note: $l = 0.02$ m, $A = 1.0 \times 10^{-5}$ m^2 .

Use the following values of the constants m and e :

$$\begin{aligned} m &= 9.1 \times 10^{-31} \text{ Kg} \\ e &= 1.6 \times 10^{-19} \text{ Coulomb} \end{aligned}$$

Choosing some value of I used during the determination of R_H , estimate the drift velocity v_d (Eq. 20.8) of the charge carriers.

Part 2

In this part of the experiment the Hall voltage as a function of sample temperature is measured, keeping the magnetic field and sample current fixed at 200 mT and 30 mA respectively. Set the circuit up as shown in Figs. 20.2a and 20.3 (lower panel). Change the axes as required in CASSYLab and enable the sample heater by pressing the heater button on the sample holder, at the same time run the program. The heater will cut off automatically at approximately 160°C. Observe the complete heating and cooling cycle.

Experiment20. The Hall Effect

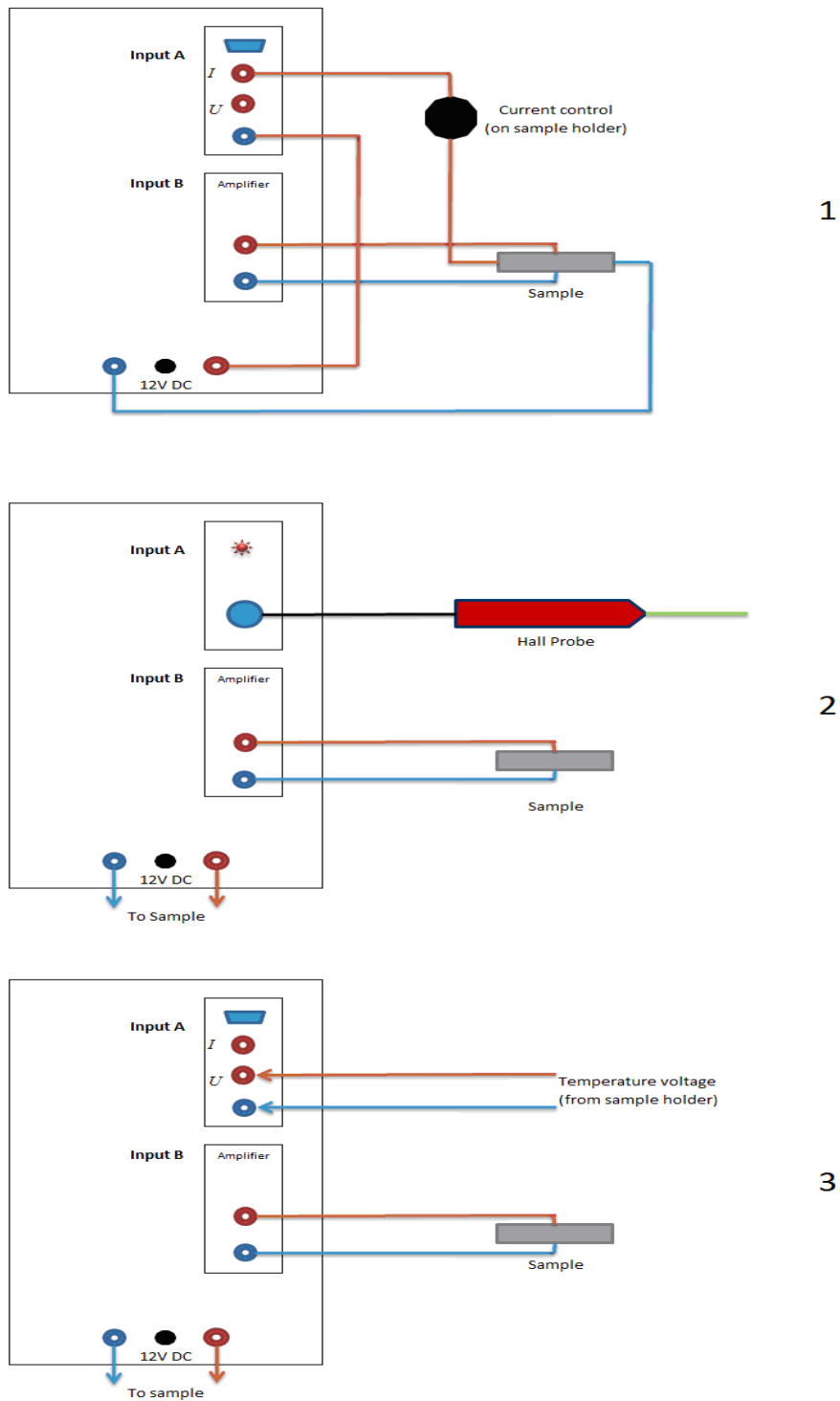


Figure 20.3: Schematic of Hall effect experiment set-up showing inputs to the CASSY interface. Diagrams 1 and 2 refer to Part 1 and diagram 3 to Part 2.

Note: A reading of 1 V from the temperature sensor corresponds to a temperature of 100 °C. The scale is linear.

Compare your graph with your neighbours on the other apparatus (if available). Do you notice any differences in your graphs? Explain.

Your graph should show, initially at least, a decrease in Hall voltage with increasing temperature. Since the measurements were made with a constant current and magnetic field this behaviour must be attributable to an increase in the number of charge carriers (see Eq. 20.17). In a semiconductor, increasing temperature makes it easier for charge carriers to jump into the conduction band and participate in conduction. From Eq. 20.8 this implies that the drift velocity of the carriers is reduced i.e. they become less mobile at higher temperatures.

Questions

1. How does your estimate of v_d compare with the thermal velocity of charge carriers at room temperature. Explain the difference in terms of the Drude model.
2. Did the sign of the Hall voltage change at a certain temperature in Part 2 of your experiment? Can you explain this in terms of the sign of the dominant charge carrier in the material at this temperature?