

Access to Science - Mathematics 1
ADEDEX424
Chapter 4 Exercises

1. For each of the following:

(i) Say whether or not it is a function and if not say why not.

(ii) If it is a function state the domain and the codomain.

(a)

$$f: \mathbb{R} \rightarrow \mathbb{R}^+ \\ x \mapsto x$$

(b)

$$f: \mathbb{R}^+ \rightarrow \mathbb{R} \\ x \mapsto x$$

(c)

$$f: \mathbb{R}^+ \rightarrow \mathbb{R} \\ x \mapsto -x^2$$

(d)

$$f: \mathbb{R} \rightarrow \mathbb{R}^+ \\ x \mapsto -x^2$$

(e) In the following, \mathbb{R}^- will denote the non-positive real numbers.

$$f: \mathbb{R}^- \rightarrow \mathbb{R}^+ \\ x \mapsto \sqrt{-x}$$

(f)

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}^- \\ x \mapsto \sqrt{x}$$

(g)

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \\ x \mapsto x - 1$$

2. Sketch the graph of each of the functions in Question 1 (including arrows on the end of the curves if necessary).

3. Sketch the graph of each of the following functions.

(a)

$$f: \{-36, -25, -16, -9, -4, 0\} \rightarrow \mathbb{R}^+ \\ x \mapsto \sqrt{-x}$$

(b)

$$f: \{-4, -2, 0, 1, 4\} \rightarrow \{0, 2, 4\} \\ -4 \mapsto 0 \\ -2 \mapsto 4 \\ 0 \mapsto 2 \\ 1 \mapsto 2 \\ 4 \mapsto 4$$

(c)

$$f: \{x \in \mathbb{R}: -1 \leq x \leq 2\} \rightarrow \{x \in \mathbb{R}: -1 \leq x \leq 7\} \\ x \mapsto 2x + 1$$

4. For each of the following functions say whether they are injective, surjective or bijective. If a function is not injective or surjective then say why not.

(a)

$$f: \{A, B, C, D, E\} \rightarrow \{1, 2, 3, 4\} \\ A \mapsto 2 \\ B \mapsto 2 \\ C \mapsto 1 \\ D \mapsto 4 \\ E \mapsto 3$$

(b)

$$f: \{A, B, C, D, E\} \rightarrow \{1, 2, 3, 4\} \\ A \mapsto 1 \\ B \mapsto 3 \\ C \mapsto 1 \\ D \mapsto 3 \\ E \mapsto 4$$

(c)

$$\begin{aligned} f: \{A, B, C, D\} &\rightarrow \{1, 2, 3, 4, 5\} \\ A &\mapsto 1 \\ B &\mapsto 2 \\ C &\mapsto 2 \\ D &\mapsto 4 \end{aligned}$$

(d)

$$\begin{aligned} f: \{A, B, C, D\} &\rightarrow \{1, 2, 3, 4, 5\} \\ A &\mapsto 3 \\ B &\mapsto 4 \\ C &\mapsto 5 \\ D &\mapsto 2 \end{aligned}$$

(e)

$$\begin{aligned} f: \{A, B, C, D\} &\rightarrow \{1, 2, 3, 4\} \\ A &\mapsto 2 \\ B &\mapsto 3 \\ C &\mapsto 1 \\ D &\mapsto 1 \end{aligned}$$

(f)

$$\begin{aligned} f: \{A, B, C, D\} &\rightarrow \{1, 2, 3, 4\} \\ A &\mapsto 2 \\ B &\mapsto 3 \\ C &\mapsto 1 \\ D &\mapsto 4 \end{aligned}$$

(g)

$$\begin{aligned} f: \mathbb{R}^- &\rightarrow \mathbb{R} \\ x &\mapsto -2x^2 \end{aligned}$$

(h)

$$\begin{aligned} f: \mathbb{R} &\rightarrow \mathbb{R}^- \\ x &\mapsto -2x^2 \end{aligned}$$

(i)

$$\begin{aligned} f: \mathbb{R}^- &\rightarrow \mathbb{R}^- \\ x &\mapsto -2x^2 \end{aligned}$$

5. For each of the bijective functions you found in Question 4, give the inverse function.

6. Figure 1 contains the graphs of four of the following functions:

- (i) $y = 3^x$.
- (ii) $y = -4^x$.
- (iii) $y = -\left(\frac{2}{3}\right)^x$.
- (iv) $y = \left(\frac{3}{4}\right)^x$.
- (v) $y = \log_3(x)$.
- (vi) $y = \log_{1/4}(x)$.

Match the functions to the graphs.

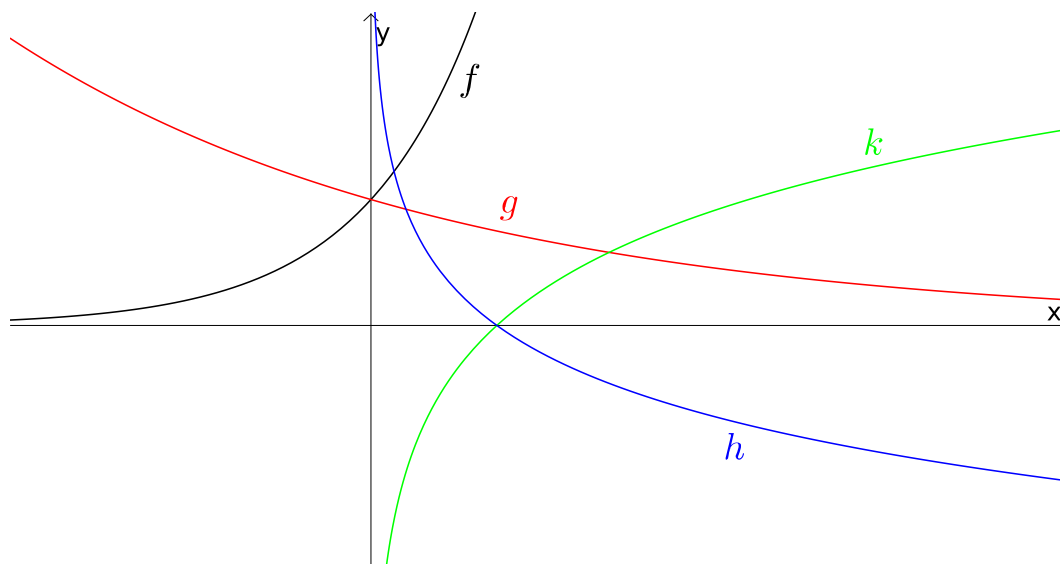


Figure 1: Various functions

7. (a) Convert the following angles to radians, leaving your answers as multiples of π .

- (i) 30° .
- (ii) 135° .
- (iii) 150° .
- (iv) 330° .

(b) Convert the following angles to degrees.

- (i) $\frac{\pi}{4}$ Radians.
- (ii) $\frac{\pi}{2}$ Radians.
- (iii) $\frac{2\pi}{3}$ Radians.

(iv) $\frac{7\pi}{4}$ Radians.

8. Without using a calculator, find the following:

(a) $\sin\left(\frac{5\pi}{6}\right)$.

(b) $\cos\left(\frac{7\pi}{6}\right)$.

(c) $\tan\left(\frac{5\pi}{4}\right)$.

(d) $\sin\left(-\frac{\pi}{4}\right)$.

(e) $\operatorname{cosec}\left(\frac{4\pi}{3}\right)$.

(f) $\cot\left(\frac{9\pi}{4}\right)$.

(g) $\cos\left(\frac{38\pi}{3}\right)$.