EXPERIMENT 11

Determination of e/m for the Electron

WARNING - Please be careful because high voltages are used in this Experiment

Introduction

The ratio of charge to mass, e/m, is a fundamental property of the electron. In the present experiment it is determined by measuring the deflection of a beam of electrons in electric and magnetic fields.

The apparatus (Fig. 11.1) consists of a large vacuum tube supported at the centre of a pair of Helmholtz coils. These are two co-axial circular coils of radius a with their planes separated by a distance a. This particular coil arrangement gives an almost uniform magnetic field over a fairly large region between the coils. The vacuum tube contains an electron gun which consists of a heated cathode emitting electrons by thermionic emission. These electrons are accelerated towards the anode and emerge through the slit in the anode as a narrow fan shaped beam of mono-energetic electrons. The narrow ribbon of electrons is intercepted by a flat mica sheet which is coated on one side with a luminescent screen to facilitate observation of the electron trajectory. There is also a graticule, marked with a centimeter scale on the sheet. The mica sheet is held at an angle of 15 degrees to the axis of the tube by two deflecting plates that are connected to a high voltage DC power supply.

When a voltage V_A, is applied to the deflecting plates, an electric field is established and the electron beam is deflected from its straight line path (no magnetic field is applied at this stage).

In Fig. 11.2 an electron passes between two charged plates of length L and separated from each other by a distance d. Since the electron is negatively charged and the electric field E is downward, a constant electrostatic force of magnitude eE acts upward on the electron. Thus as the electron travels parallel to the x axis at constant speed v_x, it accelerates upward with constant acceleration a_y. Applying Newton's second law $F = ma$ in the y direction, we find that

$$a_y = \frac{F}{m} = \frac{eE}{m} \tag{11.1}$$

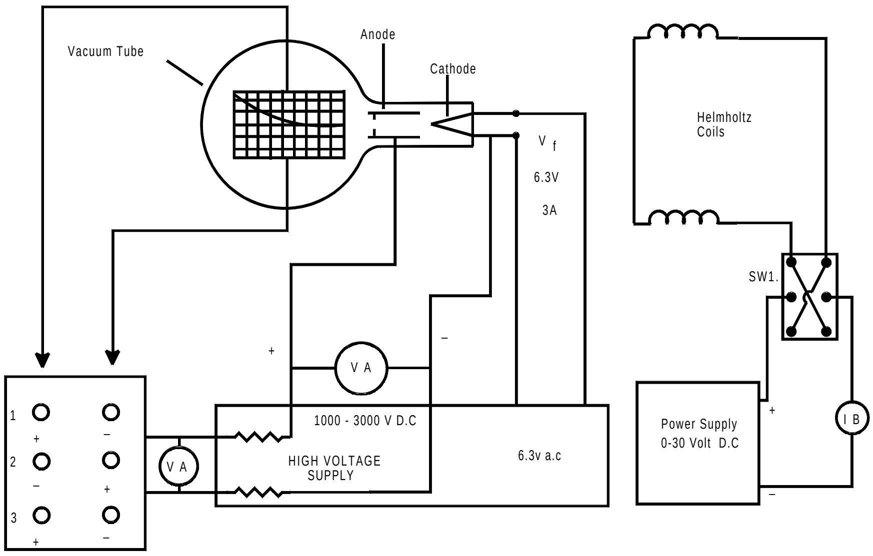

Figure 11.1: *The circuit diagram for the e/m apparatus.*

If t = time required for the electron to pass through the region between the plates, the vertical and horizontal displacements of the electron are

$$y = \frac{1}{2}a_y t^2 \text{ and } L = v_x t \tag{11.2}$$

respectively. Eliminating t between these two equations and substituting Eq. 11.1 for a_y, we find

$$y = \frac{1}{2}\left(\frac{e}{m}\right)\frac{EL^2}{{v_x}^2} = \frac{1}{2}\left(\frac{e}{m}\right)\left(\frac{V_A}{d}\right)\frac{L^2}{{v_x}^2} \text{ where } E = \frac{V_A}{d} \tag{11.3}$$

Since $y \,\alpha\, L^2$ this equation demonstrates that the electron follows a parabolic trajectory as it traverses the plates. In this equation there are three unknowns e, m and v_x. In the absence of electric and magnetic fields, the electrons will travel in a straight line after passing through the slit in the anode. When a current flows in the Helmholtz coils there is a magnetic field in the region of the electron beam.

The magnetic field B (in tesla and abbreviated as T) parallel to the axis of the coils, has a size of

$$B = \frac{8\mu_o n I_B}{\sqrt{125}a} \tag{11.4}$$

where μ_O = permeability of free space = 4 $\pi \times 10^{-7} = 1.26 \times 10^{-6}$ tesla.m/amp

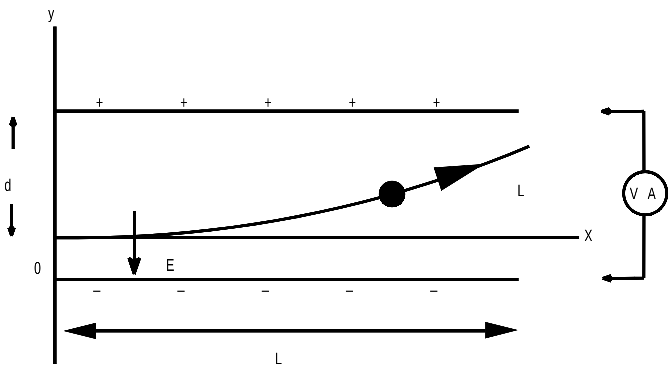

Figure 11.2: *Trajectory of an electron passing between two charged plates of length L .*

n = number of turns on each coil
I_B = coil current in amps
and a = coil radius in metres.

In this experiment n = 320 and you should measure a but note that it has an approximate value of 0.06m.

If the electron beam is perpendicular to the magnetic field, the force acting on the electrons (perpendicular both to the field and to the electron velocity) is given by

$$\text{Force} = Bev \tag{11.5}$$

This means that the electrons will travel in a circular path. If *r* is the radius of this path, then the centripetal force required to make an electron travel in a circular path of radius *r* is

$$\text{Centripetal force} = \frac{mv^2}{r} \tag{11.6}$$

This force is supplied by the magnetic field, so we can equate Eqs. 11.5 and 11.6 to give

$$\frac{mv^2}{r} = Bev$$

and

$$v = \frac{Ber}{m} \text{ or } \frac{e}{m} = \frac{v}{Br} \tag{11.7}$$

e/m can be determined if v is known.
There are two ways to determine v :

(a) The potential energy of the electrons (eV_A) can be equated to their kinetic energy $(\frac{1}{2}\, mv^2)$ i.e.

$$\frac{1}{2}\, mv^2 = eV_A \tag{11.8}$$

e/m can now be determined by substitution in Eq. 11.7 (see Eq. 11.12).

(b) Thompson showed that if an electric field of strength E is applied at the same time as, and perpendicular to, a magnetic field B′, so that the two deflections are in the same plane but in opposite directions, these can be balanced by adjustment of the fields so that

$$Ee = B'ev$$

$$\text{yielding } \; v = E/B' \tag{11.9}$$

This experiment divides naturally into four parts and you should complete them in the order given here.

Experimental Procedure

Electric Field

Set the current in the Helmholtz coils (I_B) to zero. Use the high voltage power supply (voltage 0 to 3000 V) to establish a potential difference between the two plates that support the graticule (Fig. 11.1). Note that one plate is at cathode potential or zero volts and the other plate is at the same potential as the anode (V_A). The magnitude of the electric field between the plates is given by E = V_A/d where d is the plate separation which is about 5.2 cm as measured on the centimetre scale on the graticule. The high voltage is connected to the electrodes via a box with three positions. In the first position, the upper plate is at the positive voltage, in the second position the bottom plate is at the positive voltage and in the third position the electric field is zero because both plates are at the same potential i.e. the anode potential V_A.

Set the top plate to +1000 V and note that the luminous path of the electron beam follows a parabolic curve towards the top plate.

The equation of this parabola (Eq. 11.3 with x = L and $v_x = v$) is

$$y = \frac{1}{2}\left(\frac{e}{m}\right)\left(\frac{V_A}{d}\right)\frac{x^2}{v^2}$$

where y is the vertical deflection over the distance x. The maximum value of y is $\pm$ 2.6 cm and the corresponding maximum value of x is x = 10 cm. Set V_A = +1000 V and note the values of x and y along the parabolic path of the electrons. Repeat the above measurements with V_A = -1000 V. You should also record the straight line path of the electrons when both plates are at the same potential (position 3). Plot a graph of y versus x for V_A = 1000 V and verify that y is proportional to x^2 by fitting the parabolic curve to the data.

Magnetic Deflection

Using the apparatus (Fig. 11.1) set both plates to the same potential as the anode (i.e. position 3 – no electric field) and V_A = 1000 V. The Helmholtz coils are connected to a low voltage power supply via a three position switch, A, B and C (Fig. 11.1). In position A the current flows in one direction and in position C, it is reversed. In position B there is additional circuitry to dissipate the back EMF from the Helmholtz coils when the switch is opened. **In switching from A to C or from C to A you *must* remain in position B for at least 5 seconds to dissipate the back EMF and safeguard the power supply**. Energise the Helmholtz coils and observe that the luminous beam traces out a segment of a circular path. With V_A fixed, note that the radius of the circle decreases with increasing Helmholtz coil current. With I_B fixed, note that the radius increases with increasing anode potential because of the higher velocity of the electrons. A reversal in the direction of the Helmholtz coil current I_B reverses the luminous circular path of the electron beam because of the reversal in direction of the magnetic field. The radius of the circle passing through the origin (x = 0, y = 0 which is at the exit aperture of the anode) and the points $x \pm y$ is given by

$$r = \frac{x^2 + y^2}{2y} \tag{11.10}$$

The maximum value of x is 10 cm and of y, ± 2.6 cm (as already stated).

Let V_A = 1000, 1500, 2000, 2500 and 3000 V recording the values of I_B for which the electron beam passes through the points $x = 10, y = \pm 2.6$. The value of e/m cannot be deduced from Eq. 11.7 because the electron velocity *v* is unknown but, the value of *v* can be determined if it is assumed that the electric potential energy is converted completely to electron kinetic energy,

$$eV_A = \frac{1}{2}mv^2$$

yielding

$$v^2 = \frac{2eV_A}{m} \tag{11.11}$$

Substituting this in Eq. 11.7 yields

$$\frac{e}{m} = \frac{2V_A}{B^2 r^2} \tag{11.12}$$

For each value of V_A, compute the magnetic field (B) from Eq. 11.4, the radius of the circular path (r) from Eq. 11.10 and hence determine e/m. Tabulate your results as shown in Table 11.1. Calculate the mean value obtained for e/m and the standard error on the mean.

Combined Electric and Magnetic Deflections

Thompson showed that if an electric field E is applied at the same time, and perpendicular to, a magnetic field B′ so that the two deflections are in the same plane but in opposite directions, these can be balanced when eE = B'e*v* or the velocity $v = E/B'$. For values of V_A = 1000, 1500, 2000, 2500 and 3000 V, record the values of $I_{B'}$ for balance and hence calculate B'. You have already recorded previously (magnetic deflection) the values of I_B and B (for the same values of V_A) at which the electron beams passes through the points $x = 10, y = \pm 2.6$.

Table 11.1: *B adjusted for circles to pass through the points ($x = 10, y = \pm 2.6$).*

V_A	I_B (y +)	I_B(y-)	I_B (mean)	B (T)	e/m (C/kg)	$v(ms^{-1})$
1000 V						
1500 V						
2000 V						
2500 V						
3000 V						

Combining Eqs. 11.7 and 11.9 yields

$$\frac{e}{m} = \frac{v}{Br} = \frac{E}{B'}\frac{1}{Br} = \frac{V_A}{d}\frac{1}{B'Br} \tag{11.13}$$

For each value of V_A compute e/m and tabulate your results.

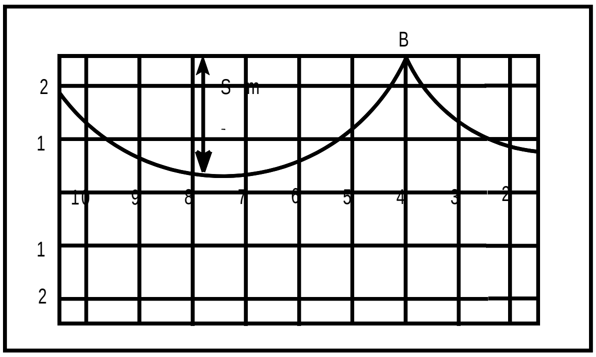

Figure 11.3: *Trochoid trajectory of the electron beam using the electron mirror set-up.*

Electron Mirror

With the electric and magnetic deflections opposite to each other, increase the current in the Helmholtz coils considerably beyond that required for balance (1.0 - 2.0 A) and observe the looped path of the electron beam as shown in Fig. 11.3. The strong magnetic field deflects the electrons along the electric field where they are decelerated and brought to rest at point B in Fig. 11.3. At

B the electrons are accelerated in the direction of the electric field and once again deflected in the magnetic field. It can be shown that the path of the electron beam is a trochoid. (A trochoid is a plane curve traced by a point on a circle or on its extended radius as the circle rolls without slipping, in a straight line: when the point is on the circumference of the circle, the curve traced is a cycloid).

It can be shown that S_m (Fig. 11.3) is given by

$$S_m = \frac{2mE}{eB^2} \quad \text{yielding} \quad \frac{e}{m} = \frac{2E}{B^2 S_m} \tag{11.14}$$

Connect the deflecting plates to the anode and cathode and adjust I_B until the trochoid in Fig. 11.3 is obtained. Measure S_m for V_A = 2000 and 3000 V and calculate e/m. For V_A = 3000 V, typical values are I_B = 1.1 A and S_m = 0.026m.

Further adjustments of V_A and I_B can produce additional spiral effects illustrating the extent to which an electron beam can be controlled. You should make drawings of these unusual configurations for V_A = 1000 V.

Please note that the accepted value for e is 1.602×10^{-19} C and m is 9.109×10^{-31} kg. The accepted value of e/m is = 1.759 $\times 10^{11}$ C kg^{-1} and you should obtain results within a factor of 2 of this value with the apparatus provided.

Questions

1. Why is the tube evacuated?
2. What is the order of magnitude of the error made in neglecting the magnetic field of the Earth in this experiment?
3. Write Eq. 11.5 in vector notation for the general case where the electron makes an angle θ with the magnetic field. In this case, what advantages has the vector notation?
4. What is meant by a back EMF?