EXPERIMENT 4

Microwave Experiments

# Introduction

Microwaves are electromagnetic radiation in the "centimeter" range of wavelengths. As such, they, like light, will exhibit typical wave properties like interference and diffraction. However, since their wavelength is typically 50,000 times that of light, it is much easier to demonstrate these typical wave properties using microwaves. As an example, the distance between successive maxima in a standing wave pattern is 1.5 cm for 3 cm microwaves.

The experiment has three parts. The first uses a double slit (like Young's Slits) arrangement to give an estimate of the microwave wavelength, and in the second part, a Michelson interferometer is used to give a more reliable measure of the wavelength. The third part of the experiment demonstrates the technique of crystal diffraction, using a "crystal" made up of a regular array of ball bearings embedded in a polystyrene holder. All three parts use the same basic equipment of microwave source, wax lenses and receiver. The source (transmitter Tx) is a Gunn diode, which generates accurately monochromatic microwaves. These are directed from the transmitter horn, through a wax lens (to collimate the beam), and are ultimately collected, via a second such lens, at the microwave horn of the receiver (Rx), a silicon crystal microwave diode rectifier. The receiver is connected to a microammeter.

The transmitter, receiver and wax lenses are mounted on the moveable arms of a turntable, as shown below (Fig. 4.1), so that the angle between transmitted and received microwave beams can easily be measured.

# **Experimental Procedure**

## Part 1 : Double Slit

The double slit is a pair of 2 cm wide slits with a 6 cm separation, cut in a metal sheet. The set-up to be used is shown (Fig. 4.2), with the double slit at the centre of the turntable. With the transmitter and receiver facing one another through the slits, the shunt on the microammeter should be adjusted



Figure 4.1: General layout for the microwave system

so that a large scale deflection is obtained. The interference pattern may then be explored by moving the receiver arm, whilst keeping the transmitter arm fixed. A graph of intensity (current) versus  $\theta$  should be plotted, and an estimate made of the microwave wavelength by measuring the angular separation of adjacent maxima in the interference pattern and the separation of the two slits.

### Part 2 : Michelson Interferometer

The interferometer can be set up as shown below (Fig. 4.3). The mirrors are metal sheets and the beamsplitter is a hardboard sheet. The metal sheets should be carefully positioned to be perpendicular to the microwave beam, and the beamsplitter positioned at the centre of the turntable and at  $45^0$  to the beam. Any slight change in the path length of either of the beams leaving the beamsplitter changes the interference pattern at the receiver. One of the metal sheets is mounted in a frame which slides along rails. If this sheet is slowly moved, maxima and minima of current will be recorded by the receiver and a plot of "intensity" against distance can be made. The distance between successive maxima (or minima) is one half of the microwave wavelength which can thus be measured.

### Part 3 : "Crystal" Diffraction

The investigation of crystal structures can be carried out using X-rays because the interatomic spacing in crystals is typically  $10^{-10}$  m (1Å) which is of the same order of magnitude as X-ray wavelengths (see X-Ray Experiments notes in this handbook for further details). A regular array of atoms, such as is found in crystal structures, acts then as a three-dimensional diffraction grating



Figure 4.2: Young's Slits arrangement



Figure 4.3: Michelson interferometer arrangement



Figure 4.4: Bragg Scattering from a Crystal

and the condition for a diffraction maximum can be shown (see Fig. 4.4) to be

$$2d\sin\theta = n\lambda\tag{4.1}$$

where d is the spacing of the crystal planes and  $\theta$ , n and  $\lambda$  are defined in the usual way.

All the essential features of an X-ray diffraction experiment to investigate crystal structure can be reproduced in an analogue experiment, using microwaves instead of X-rays and a scaled-up version of a crystal which has aluminum spheres ("atoms"), embedded in a polystyrene block (transparent to microwaves). The set up, as shown in Fig. 4.5, is similar to the classic Bragg spectrometer used in X-ray diffraction experiments.

The "crystal" planes, to be investigated here, are the 010, 100, 110 and 210 planes (see Appendix). The Tx and Rx should first be lined up facing one another and the "crystal" positioned for investigation of the spacing of the 010 planes. The Tx and Rx arms should then be moved towards one another always keeping the angles of incidence and reflection (Fig. 4.4) equal to one another. In the region of interest  $(10^0 \le \theta \le 60^0)$  readings of detector current (relative intensity) as a function of  $\theta$  should be obtained by incrementing  $\theta$  in 2<sup>0</sup> (or smaller) intervals and noting the corresponding meter reading. A graph of relative intensity  $vs \theta$  will give the positions of first and second order reflections from the 010 planes. The crystal plane spacing  $d_{100}$  can then be obtained from Eq. 4.1 using the microwave wavelength determined in Part 2 of the experiment Michelson Interferometer. This process should be repeated for the 100 planes and  $d_{010}$  evaluated.

Knowing  $d_{010}$  and  $d_{100}$ , it is easy to calculate the position of the crystal required to give reflections from the 110 and 210 planes so that  $d_{110}$  and  $d_{210}$  can also be evaluated.



Figure 4.5: Crystal diffraction arrangement

### **Questions:**

- 1. Comment on the advantages of using microwaves in experiments such as the ones you have performed here.
- 2. Can you make any further comment on the intensity distributions obtained in Parts 1 and 2 of the experiment?
- 3. Have your microwave measurements given you any information on the z spacing of the "crystal"?
- 4. On what order would it be most logical, experimentally, to determine  $d_{210}$ ,  $d_{110}$ ,  $d_{010}$ ,  $d_{100}$ ?

# Appendix

X-rays were discovered in 1895 by Roentgen. It was found that they had great penetrating power, and were not deflected by electric or magnetic fields, which showed that they were uncharged particles. It was thought that X-rays might be electromagnetic radiation similar to light, but with a much shorter wavelength. The existence of wave motion can be definitely established only by the observation of diffraction and interference phenomena. The conditions under which these occur in the case of light is well known. To produce a diffraction pattern, the width of the aperture used must be of the same order as the wavelength of the radiation. Using the smallest apertures possible, no diffraction effects were observed using X-rays which indicated that, if they were electromagnetic waves, their wavelength was very small. In 1912 M. von Laue suggested that a crystal might be used to diffract X-rays since it was believed that the atoms in a crystal were arranged in an orderly



Figure 4.6: Ordering of atoms in crystal structures

manner and that the spacing between the atoms was of the order of 1Å, so that the crystal would act as a three dimensional grating. This was verified by Friedrick and Knipping a short time later when they produced a diffraction pattern on passing X-rays through a thin crystal of zinc blende. This was a very significant experiment since it established the wave nature of X-rays, and also confirmed the supposition that atoms in crystals are arranged in a regular pattern.

In 1912 Bragg derived a relation which explained X-ray diffraction effects in terms of reflection from atomic planes. The atoms in a crystal are arranged in an orderly way such as in Fig. 4.6, where a small section of a crystal is shown. The x, y and z axes shown are suitable for describing this crystal and the repeat distance along these axes are a, b and c respectively. The repeat distance in any direction is the smallest distance between atoms in that direction. It is easy to see that we can pick out sets of equi-spaced parallel planes that are rich in atoms, e.g. planes parallel to ABCD, ABEF or EFCD etc. The Miller Indices method of notation is used to identify the various sets of planes. The Miller indices of a plane are the reciprocals of the intercepts of the plane on the axes, in units of the corresponding repeat distances. They are denoted by (hkl), and are so chosen that they are integers, with no common factor. The intercepts of the plane ABCD are  $(2, \infty, \infty)$  and so the Miller indices are (1,0,0). The intercepts of the plane MGN are (3,2,1), the reciprocal of these intercepts are (1/3, 1/2, 1) and so the Miller indices are (2,3,6). The perpendicular distance between the planes is denoted by  $d_{hkl}$ . Since the volume density of atoms is uniform throughout the crystals, the density of atoms per plane decreases as  $d_{hkl}$ . Bragg assumed that the diffracted beams were produced by X-rays which were scattered from families of planes rich in atoms. In order to get the directions of the diffraction maxima, he assumed that each member of a family of parallel planes will act as if it were a plane mirror. Consider Fig. 4.4 which shows the side view of a crystal whose z - axis is perpendicular to the plane of the paper, so that this plane of atoms is repeated into the paper at regular intervals. The lines shown are side views of a family of planes rich in atoms. If a collimated beam of monochromatic X-rays is incident on the crystals, detailed analysis shows that we can regard the X-rays as being partially transmitted and partially reflected at each plane. The reflected X-rays from successive planes will give a diffraction maximum if they are in phase, that is if

$$CB + BD = n\lambda$$

where n is an integer. From Fig. 4.4 it is seen that

$$CB = BD = d\sin\theta$$

Therefore the condition for a diffraction maximum is

$$2d\sin\theta = n\lambda$$

This is known as Bragg's Equation and gives the relationship that must be satisfied by the spacing, d, of the crystal planes, the angle of reflection,  $\theta$ , from these planes, and the wavelength of the X-rays. The integer n gives the order of interference, i.e. n is the path difference between waves reflected from adjacent planes.

### Analysis of crystals

Bragg's equation can be used to find the perpendicular distance between adjacent planes of a family of planes. The arrangement shown in Fig. 4.4 is used. The orientation of some of the families of planes of a crystal is known, because every possible face of a crystal is parallel to a family of planes. Therefore the crystal X-ray source and detector can be set up so that the family of planes to be investigated is in a position to reflect X-rays from the source into the detector. The position of the source and detector are varied keeping the angle of incidence equal to the angle of reflection. The detector readings are plotted against  $\theta$ , giving a graph similar to Fig. 4.7. The peaks at  $\theta_1$  and  $\theta_2$  correspond to the first and second order reflections, respectively, and Bragg's equation can be written as

First order reflection $2d \sin\theta_1 = \lambda$ Second order reflection $2d \sin\theta_2 = 2 \lambda$ 

If  $\lambda$  is known, d can be calculated from these equations. It should also be noted that if d is known,  $\lambda$  can be calculated i.e. the arrangement in Fig. 4.5 with a crystal of known spacing could be used to find the wavelength of X-rays. This arrangement was first used by Bragg and is known as a Bragg spectrometer.

## **Unit Cell**

The parallelepiped - shaped volume which, reproduced by close packing in three dimensions, gives the whole crystal called the unit cell (see Fig. 4.6). It is worth noting that the unit cell may not be an entity which can be uniquely defined. One usually chooses the unit cell which displays the highest possible symmetry, for this indicates, far more clearly, the symmetry of the underlying structure. The lengths of the edges of the unit cell are generally designated a, b, c and the interaxial angles by  $\alpha, \beta$ , and  $\gamma$ , with  $\alpha$  between b and c, with  $\beta$  between c and a, and with  $\gamma$  between a and b.



Figure 4.7: Typical output from Bragg spectrometer set-up

### References

- 1. E.P. 2.1
- 2. Optics: Hecht and Zajac
- 3. Physics of the atom: Wehr Richards
- 4. Elements of Solid State Physics : Rudden & Wilson

### WWW:

- Lecture notes on diffraction http://www.abdn.ac.uk/~che241/y3-2002/3lec3.ppt
- Microwaves and mobile phones http://www.seernnadivad.org/Why.....htm